CHAPTER 2

Number Systems

(Solutions to Review Questions and Problems)

Review Questions

Q2-1. A number system shows how a number can be represented using distinct symbols.

Q2-3. The base (or radix) is the total number of symbols used in a positional number system.

Q2-5. The binary system is a positional number system that uses two symbols (0 and 1) to represent a number. The word binary is derived from the Latin root *bini* (two by two) or *binarius* (related to two). In the binary system, the base is 2.

Q2-7. The hexadecimal system is a positional number system with sixteen symbols. The word hexadecimal is derived from the Greek root *hex* (six) and the Latin root *decem* (ten). To be consistent with decimal and binary, it should have been called *sexadecimal*, from Latin roots *sex* and *decem*. In the hexadecimal system, the base is 16.

Q2-9. Four bits in binary is one hexadecimal digit.

Problems

P2-1.

a.

Place values		16		8		4		2		1		
$(01101)_2$	=	0	+	8	+	4	+	0	+	1	=	13

b.

Place values		64		32		16		8		4		2		1		
$(1011000)_2$	=	64	+	0	+	16	+	8	+	0	+	0	+	0	=	88

c.

Place values		32		16		8		4		2		1		1/2		1/4		
$(011110.01)_2$	=	0	+	16	+	8	+	4	+	2	+	0	+	0	+	1/4	=	30.25

d.

Place values		32		16		8		4		2		1		1/2		1/4		1/8		
$(111111.111)_2$	=	32	+	16	+	8	+	4	+	2	+	1	+	1/2	+	1/4	+	1/8	=	63.875

P2-3.

a.

Place values		64		8		1		
$(237)_8$	=	2×64	+	3×8	+	7×1	=	159

b.

Place values		512		64		8		1		
$(2731)_8$	=	2×512	+	7×64	+	3×8	+	1×1	=	1497

c.

Place values		512		64		8		1		1/8		
$(617.7)_8$	=		+	6×64	+	1×8	+	7×1	+	$7 \times 1/8$	=	399.875

d.

Place values		8		1		1/8		1/64		
$(21.11)_8$	=	2×8	+	1×1	+	$1 \times 1/8$	+	$1 \times 1/64$	$\approx$	17.141

P2-5.

a. 1156 = $(2204)_8$ as shown below:

0	←	2	←	18	←	144	←	**1156**
		↓		↓		↓		↓
		2		2		0		4

b. 99 = $(134)_8$ as shown below:

0	←	1	←	12	←	**99**
		↓		↓		↓
		1		4		3

c. 11.4 = $(13.3146)_8$ as shown below:

0	←	1	←	**11**		**.4**	→	.2	→	.6	→	.8	→	4
		↓		↓		↓		↓		↓		↓		
		1		3	•	3		1		4		6		

d. 72.8 = $(110.6314)_8$ as shown below:

0	←	1	←	9	←	**72**		**.8**	→	.4	→	.2	→	.6	→	.8
		↓		↓		↓		↓		↓		↓		↓		
		1		1		0	•	6		3		1		4		

P2-7.

a.

Change octal to binary					=	Change binary to hexadecimal				
$(514)_8$	=	101	001	100	=	1	0100	1100	=	$(14C)_{16}$

b.

Change octal to binary					=	Change binary to hexadecimal				
$(411)_8$	=	100	001	001	=	1	0000	1001	=	$(109)_{16}$

c.

Change octal to binary						=	Change binary to hexadecimal					
$(13.7)_8$	=	001	111	•	111	=	00	1011	•	1110	=	$(B.E)_{16}$

d.

Change octal to binary						=	Change binary to hexadecimal				
$(1256)_8$	=	001	010	101	110	=	0010	0101	1110	=	$(25E)_{16}$

P2-9.

a.

$(01101)_2$	=	001	101	=	$\mathbf{(15)_8}$

b.

$(1011000)_2$	=	001	011	000	=	$\mathbf{(130)_8}$

c.

$(011110.01)_2$	=	011	110	•	010	=	$\mathbf{(36.2)_8}$

d.

$(111111.111)_2$	=	111	111	•	111	=	$\mathbf{(77.7)_8}$

P2-11.

a.

121	=	0	+	64	+	32	+	16	+	8	+	0	+	0	+	1	=	$\mathbf{(01111001)_2}$

b.

78	=	0	+	64	+	0	+	0	+	8	+	4	+	2	+	0	=	$\mathbf{(01001110)_2}$

c.

255	=	128	+	64	+	32	+	16	+	8	+	4	+	2	+	1	=	$\mathbf{(11111111)_2}$

d.

214	=	128	+	64	+	0	+	16	+	0	+	4	+	2	+	0	=	$\mathbf{(11010110)_2}$

P2-13.

- **a.** binary: $2^6 - 1 = 63$
- **b.** decimal: $10^6 - 1 = 999{,}999$
- **c.** hexadecimal: $16^6 - 1 = 16{,}777{,}215$
- **d.** octal: $8^6 - 1 = 262{,}143$

P2-15.

- **a.** $\lceil 5 \times (\log 2) / (\log 10) \rceil = \lceil 16.6 \rceil = 2$
- **b.** $\lceil 3 \times (\log 8) / (\log 10) \rceil = \lceil 16.6 \rceil = 3$
- **c.** $\lceil 3 \times (\log 16) / (\log 10) \rceil = \lceil 16.6 \rceil = 4$

P2-17. Using the result of previous exercise, we can find the equivalent as:

- **a.** $7.1875 = (111)_2 + (0.001)_2 + (0.0001)_2 = (111.0011)_2$
- **b.** $12.540625 = (1100)_2 + (0.1)_2 + (0.001)_2 + (0.000001)_2 = (1100.101001)_2$

c. $11.40625 = (1011)_2 + (0.01)_2 + (0.001)_2 + (0.00001)_2 = (1011.01101)_2$

d. $0.375 = (0.01)_2 + (0.001)_2 = (0.011)_2$

P2-19.

a. $\lceil \log_2 1000 \rceil = \lceil \log 1000 / \log 2 \rceil = \lceil 9.97 \rceil = 10$

b. $\lceil \log_2 100{,}000 \rceil = \lceil \log 100{,}000 / \log 2 \rceil = \lceil 16.6 \rceil = 17$

c. $\lceil \log_2 64 \rceil = \lceil \log_2 2^6 \rceil = \lceil 6 \times \log_2 2 \rceil = \lceil 6 \rceil = 6$

d. $\lceil \log_2 256 \rceil = \lceil \log_2 2^8 \rceil = \lceil 8 \times \log_2 2 \rceil = \lceil 8 \rceil = 8$

P2-21.

a.

17×256^3	+	234×256^2	+	34×256^1	+	14×256^0	=	**300,556,814**

b.

14×256^3	+	56×256^2	+	234×256^1	+	56×256^0	=	**238,611,000**

c.

110×256^3	+	14×256^2	+	56×256^1	+	78×256^0	=	**1,864,425,678**

d.

24×256^3	+	56×256^2	+	13×256^1	+	11×256^0	=	**406,326,539**

P2-23.

- **a.** 15
- **b.** 27
- **c.** This is not a valid Roman Numeral (V cannot come before L)
- **d.** 1157

P2-25.

- **a.** Not valid because I cannot come before M
- **b.** Not valid because I cannot come before C
- **c.** Not valid because V cannot come before C
- **d.** Not valid because 5 is written as V not VX

P2-27.

a. First, we convert the three numbers to base 60 as shown below:

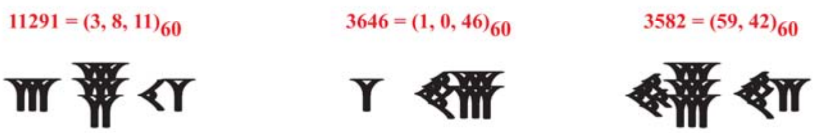
0 ←	3 ←	188 ←	**11291**
	↓	↓	↓
	3	8	11

0 ←	1 ←	60 ←	**3646**
	↓	↓	↓
	1	0	46

0 ←	59 ←	**3582**
	↓	↓
	59	42

b. The equivalent Babylonian numerals are shown in Figure P2-27.

Figure P2-27 *Babylonian numerals*



c. In Babylonian numerals, they used extra space when a zero was needed in the middle of the number. When a zero was need at left, they did not use anything; They probably recognized it from the context.