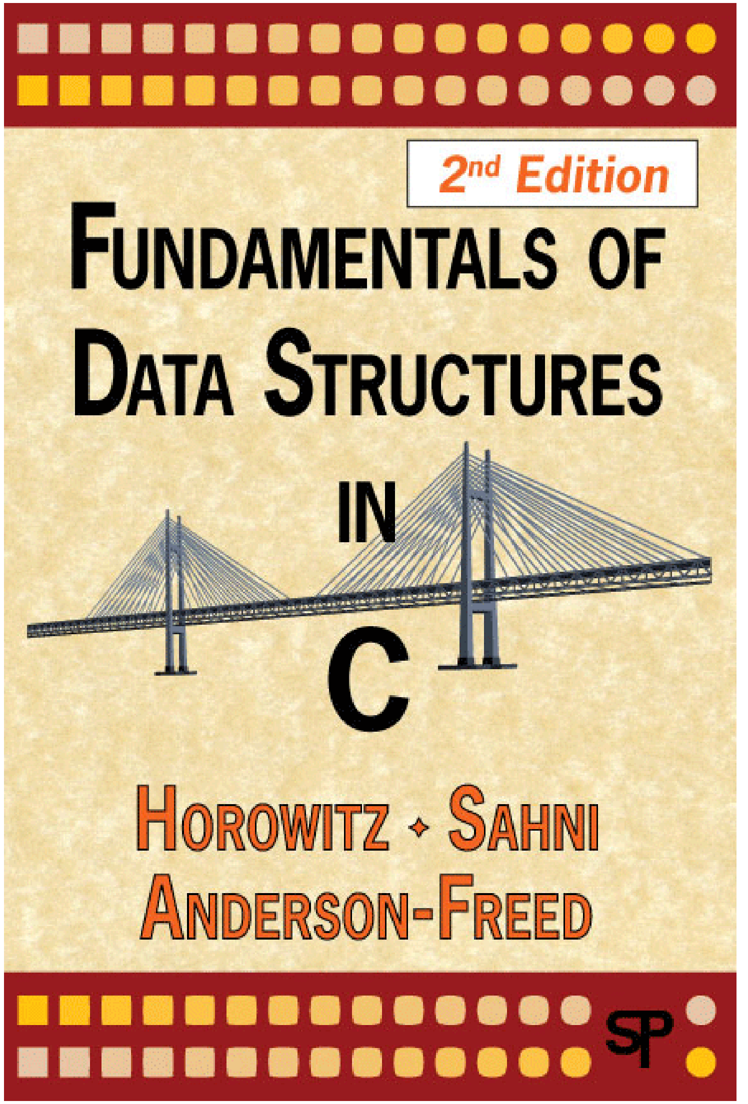
Data Structures

Books

Fundamentals of Data Structures in C, 2nd Edition.

(開發圖書，(02) 8242-3988)



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  - Quiz 20 pts
  - Computer-based Test 20 pts
  - Homework 20 pts
  - Midterm Exam 25 pts
  - Final Exam 25 pts
  - Roll Call 3 times

Introductory

- Raise your hand is always welcome!
- No phone, walk, sleep, and late during the lecture time.
- Data structure is not the fundamental course for programming.
- Slides are not enough. To master the materials, page-by-page reading is necessary.

Outline

- Basic Concept
- Arrays and Structures
- Stacks and Queues
- Linked Lists
- Trees
- Graphs
- Sorting
- Hashing

Midterm: Basic Concept, Arrays and Structures, Stacks and Queues, Linked Lists, Trees

Final: Trees, Graphs, Sorting, Hashing

CHAPTER 1

BASIC CONCEPT

All the programs in this file are selected from

Ellis Horowitz, Sartaj Sahni, and Susan Anderson-Freed
"Fundamentals of Data Structures in C",

Algorithm

- Definition
  An *algorithm* is a finite set of instructions that accomplishes a particular task.
- Criteria
  - input
  - output
  - definiteness: clear and unambiguous
  - finiteness: terminate after a finite number of steps
  - effectiveness: instruction is basic enough to be carried out

Data Type

- Data Type
  A data type is a collection of *objects* and a set of *operations* that act on those objects.
- Abstract Data Type (ADT)
  An *ADT* is a data type that is organized in such a way that **the specification of the objects and the operations on the objects** is separated from
  - the representation of the objects .
  - the implementation of the operations.

Specification vs. Implementation

- Operation specification
  - function name
  - the types of arguments
  - the type of the results
- Implementation independent

***Structure 1.1:** Abstract data type *Natural_Number*

structure Natural_Number is

representation

objects: an ordered subrange of the integers starting at zero and ending at the maximum integer (*INT_MAX*) on the computer

functions:

for all x, y ∈ *Nat_Number*; *TRUE*, *FALSE* ∈ *Boolean*
and where +, -, <, and == are the usual integer operations.

Nat_Num Zero ( ) ::= 0
Boolean Is_Zero(x) ::= **if** (x) **return** *FALSE*
  else return *TRUE*
Nat_Num Add(x, y) ::= **if** ((x+y) <= *INT_MAX*) **return** x+y
  else return *INT_MAX*
Boolean Equal(x,y) ::= **if** (x== y) **return** *TRUE*
  else return *FALSE*
Nat_Num Successor(x) ::= **if** (x == *INT_MAX*) **return** x
  else return x+1
Nat_Num Subtract(x,y) ::= **if** (x<y) **return** 0
  else return x-y

implementation

end *Natural_Number*

::= is defined as

Measurements

- Criteria
  - Is it correct?
  - Is it readable?
  - …
- Performance Measurement (machine dependent)

- Performance Analysis (machine independent)
  - space complexity: storage requirement
  - time complexity: computing time

Space Complexity
$S(P)=C+S_P(I)$

- Fixed Space Requirements (C)
  Independent of the characteristics of the inputs and outputs
  - instruction space
  - space for simple variables, fixed-size structured variable, constants
- Variable Space Requirements ($S_P(I)$)
  depend on the instance characteristic I
  - number, size, values of inputs and outputs associated with I
  - recursive stack space, formal parameters, local variables, return address

***Program 1.10:** Simple arithmetic function

```
float abc(float a, float b, float c)
{
   return a + b + b * c + (a + b - c) / (a + b) + 4.00;
 }
```

This function has only fixed space requirements $S_{abc}(I) = 0$

***Program 1.11:** Iterative function for summing a list of numbers

```
float sum(float list[ ], int n)
{
 float tempsum = 0;
 int i;
 for (i = 0; i<n; i++)
    tempsum += list [i];
 return tempsum;
}
```

$S_{sum}(I) = 0$

Recall: pass the address of the first element of the array & pass by value

*__Program 1.12:__ Recursive function for summing a list of numbers

```
float rsum(float list[ ], int n)
{
  if (n) return rsum(list, n-1) + list[n-1];
  return 0;
}
```

$S_{sum}(I)=S_{sum}(n)=12n$

Assumptions:

*__Figure 1.1:__ Space needed for one recursive call of Program 1.12

Type	Name	Number of bytes
parameter: array pointer parameter: integer return address:(used internally)	list [ ] n	4 4 4 (unless a far address)
TOTAL per recursive call		12

Time Complexity

$$T(P)=C+T_P(I)$$

- C: Compile time
  independent of instance characteristics
- T_P : Run (execution) time
- Definition

  $T_P(n)=c_aADD(n)+c_sSUB(n)+c_lLDA(n)+c_{st}STA(n)$

  A *program step* is a syntactically or semantically meaningful program segment whose execution time is independent of the instance characteristics.
- Example
  - abc = a + b + b * c + (a + b - c) / (a + b) + 4.0
  - abc = a + b + c

- Regard as the same unit
- machine independent

Methods to compute the step count

1. Introduce variable count into programs
2. Tabular method
   - Determine the total number of steps contributed by each statement
     step per execution × frequency
   - add up the contribution of all statements

Tabular Method

***Figure 1.2:** Step count table for Program 1.11*

Iterative function to sum a list of numbers

steps/execution

Statement	s/e	Frequency	Total steps
float sum(float list[ ], int n)	0	0	0
{	0	0	0
float tempsum = 0;	1	1	1
int i;	0	0	0
for(i=0; i <n; i++)	1	n+1	n+1
tempsum += list[i];	1	n	n
return tempsum;	1	1	1
}	0	0	0
Total			2n+3

Iterative summing of a list of numbers

***Program 1.13:** Program 1.11 with **count statements**

```
float sum(float list[ ], int n)
{
   float tempsum = 0; count++; /* for assignment */
   int i;
   for (i = 0; i < n; i++) {
        count++;            /*for the for loop */
        tempsum += list[i]; count++;  /* for assignment */
   }
   count++;         /* last execution of for */
   count++;         /* for return */
   return tempsum;

}
```

2n + 3 steps

Program 1.13

initial
n = 3,   count = 0

count = 1

```
float sum(float list[], int n) {
    float tempsum = 0;
    int i;
    for(i = 0; i < n; i++){
        tempsum += list[i];
    }
    return tempsum;
}
```

Program 1.13

initial
n = 3, count = 0

```
float sum(float list[], int n) {
    float tempsum = 0;
    int i;
    for(i = 0; i < n; i++){
        tempsum += list[i];
    }
    return tempsum;
}
```

i = 0, count = 2

Program 1.13

initial
n = 3, count = 0

```
float sum(float list[], int n) {
    float tempsum = 0;
    int i;
    for(i = 0; i < n; i++){
        tempsum += list[i];
    }
    return tempsum;
}
```

count = 3

Program 1.13

initial
n = 3,   count = 0

```
float sum(float list[], int n) {
    float tempsum = 0;
    int i;
    for(i = 0; i < n; i++){
        tempsum += list[i];
    }
    return tempsum;
}
```

i = 1, count = 4

Program 1.13

initial
n = 3, count = 0

```
float sum(float list[], int n) {
    float tempsum = 0;
    int i;
    for(i = 0; i < n; i++){
        tempsum += list[i];                count = 5
    }
    return tempsum;
}
```

Program 1.13

initial
n = 3, count = 0

i = 2, count = 6

```
float sum(float list[], int n) {
    float tempsum = 0;
    int i;
    for(i = 0; i < n; i++){
        tempsum += list[i];
    }
    return tempsum;
}
```

Program 1.13

initial
n = 3,   count = 0

```
float sum(float list[], int n) {
    float tempsum = 0;
    int i;
    for(i = 0; i < n; i++){
        tempsum += list[i];
    }
    return tempsum;
}
```

count = 7

Program 1.13

initial
n = 3,   count = 0

```
float sum(float list[], int n) {
    float tempsum = 0;
    int i;
    for(i = 0; i < n; i++){
        tempsum += list[i];
    }
    return tempsum;
}
```

i = 3, count = 8

Program 1.13

initial
n = 3,   count = 0

```
float sum(float list[], int n) {
    float tempsum = 0;
    int i;
    for(i = 0; i < n; i++){
        tempsum += list[i];
    }
    return tempsum;
}
```

count = 9

回outline

Program 1.13

initial
n = 3,   count = 0

```
float sum(float list[], int n) {
    float tempsum = 0;              1次
    int i;
    for(i = 0; i < n; i++){         n+1次
        tempsum += list[i];         n次
    }
    return tempsum;             +   1次
}                               ----------
                                    2n+3次
```

Recursive summing of a list of numbers

***Program 1.15:** Program 1.12 with count statements added

```
float rsum(float list[ ], int n)
{
        count++;      /*for if conditional */
        if (n) {
                count++;  /* for return and rsum invocation */
                return rsum(list, n-1) + list[n-1];
        }
        count++;
        return list[0];
}
```

2n+2

Program 1.15

initial
n = 3,   count = 0

```
float rsum(int list[], int n){
    if(n){
        return rsum(list, n-1) + list[n-1];
    }
    return list[0];
}
```

n=3, count = 1

Program 1.15

initial
n = 3,   count = 0

```
float rsum(int list[], int n){
    if(n){
        return rsum(list, n-1) + list[n-1];
    }
    return list[0];
}
```

n=3, count = 2

Program 1.15

initial
n = 3,   count = 0

```
float rsum(int list[], int n){
    if(n){
        return rsum(list, n-1) + list[n-1];
    }
    return list[0];
}
```

n=2, count = 3

Program 1.15

initial
n = 3,   count = 0

```
float rsum(int list[], int n){
    if(n){
        return rsum(list, n-1) + list[n-1];
    }
    return list[0];
}
```

n=2, count = 4

Program 1.15

initial
n = 3,   count = 0

```
float rsum(int list[], int n){
    if(n){
        return rsum(list, n-1) + list[n-1];
    }
    return list[0];
}
```

n=1, count = 5

Program 1.15

initial
n = 3,   count = 0

```
float rsum(int list[], int n){
    if(n){
        return rsum(list, n-1) + list[n-1];
    }
    return list[0];
}
```

n=1, count = 6

Program 1.15

initial
n = 3, count = 0

```
float rsum(int list[], int n){
    if(n){
        return rsum(list, n-1) + list[n-1];
    }
    return list[0];
}
```

n=0, count = 7

Program 1.15

initial
n = 3, count = 0

```
float rsum(int list[], int n){
    if(n){
        return rsum(list, n-1) + list[n-1];
    }
    return list[0];
}
```

n=0, count = 8

Program 1.15

initial

n = 3,   count = 0

```
float rsum(int list[], int n){
    if(n){
        return rsum(list, n-1) + list[n-1];
    }
    return list[0];
}
```

n+1次

n次

\+ 1次

2n+2次

Recursive Function to sum of a list of numbers

***Figure 1.3:** Step count table for recursive summing function

Statement	s/e	Frequency	Total steps
float rsum(float list[ ], int n)	0	0	0
{	0	0	0
if (n)	1	n+1	n+1
return rsum(list, n-1)+list[n-1];	1	n	n
return list[0];	1	1	1
}	0	0	0
Total			2n+2

Matrix addition

***Program 1.16:** Matrix addition*

```
void add(int a[ ] [MAX_SIZE], int b[ ] [MAX_SIZE],
                          int c [ ] [MAX_SIZE], int rows, int cols)
{
    int i, j;
    for (i = 0; i < rows; i++)
       for (j= 0; j < cols; j++)
         c[i][j] = a[i][j] +b[i][j];                          rows * cols
 }
```

Matrix Addition

***Figure 1.4:** Step count table for matrix addition*

Statement	s/e	Frequency	Total steps
Void add (int a[ ][MAX_SIZE] • • •)	0	0	0
{	0	0	0
int i, j;	0	0	0
for (i = 0; i < row; i++)	1	rows+1	rows+1
for (j=0; j< cols; j++)	1	rows • (cols+1)	rows • cols+rows
c[i][j] = a[i][j] + b[i][j];	1	rows • cols	rows • cols
}	0	0	0
Total			2rows • cols+2rows+1

***Program 1.17:** Matrix addition with count statements

```
void add(int a[ ][MAX_SIZE], int b[ ][MAX_SIZE],
                         int c[ ][MAX_SIZE], int row, int cols )
{
   int i, j;
   for (i = 0; i < rows; i++){
        count++; /* for i for loop */
        for (j = 0; j < cols; j++) {
           count++; /* for j for loop */
           c[i][j] = a[i][j] + b[i][j];
           count++; /* for assignment statement */
        }
        count++;    /* last time of j for loop */
  }
  count++;         /* last time of i for loop */
}
```

2*rows * cols + 2 rows + 1

Program 1.17

initial
rows=2, cols=3, count = 0

i = 0, count = 1

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
            c[][MAX_SIZE], int rows, int cols) {
    int i, j;
    for(i = 0;i < rows; i++){
        for(j = 0;j < cols; j++){
            c[i][j] = a[i][j] + b[i][j];
        }
    }
}
```

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
              c[][MAX_SIZE], int rows, int cols) {
    int i, j;
    for(i = 0;i < rows; i++){
        for(j = 0;j < cols; j++){
            c[i][j] = a[i][j] + b[i][j];
        }
    }
}
```

j = 0, count = 2

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
              c[][MAX_SIZE], int rows, int cols) {
    int i, j;
    for(i = 0;i < rows; i++){
        for(j = 0;j < cols; j++){
            c[i][j] = a[i][j] + b[i][j];
        }
    }
}
```

count = 3

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
              c[][MAX_SIZE], int rows, int cols) {
    int i, j;
    for(i = 0;i < rows; i++){
        for(j = 0;j < cols; j++){
            c[i][j] = a[i][j] + b[i][j];
        }
    }
}
```

j = 1, count = 4

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
           c[][MAX_SIZE], int rows, int cols) {
   int i, j;
   for(i = 0;i < rows; i++){
      for(j = 0;j < cols; j++){
         c[i][j] = a[i][j] + b[i][j];
      }
   }
}
```

count = 5

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
            c[][MAX_SIZE], int rows, int cols) {
    int i, j;
    for(i = 0;i < rows; i++){
        for(j = 0;j < cols; j++){
            c[i][j] = a[i][j] + b[i][j];
        }
    }
}
```

j = 2, count = 6

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
            c[][MAX_SIZE], int rows, int cols) {
   int i, j;
   for(i = 0;i < rows; i++){
      for(j = 0;j < cols; j++){
         c[i][j] = a[i][j] + b[i][j];
      }
   }
}
```

count = 7

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
              c[][MAX_SIZE], int rows, int cols) {
    int i, j;
    for(i = 0;i < rows; i++){
        for(j = 0;j < cols; j++){
            c[i][j] = a[i][j] + b[i][j];
        }
    }
}
```

j = 3, count = 8

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
            c[][MAX_SIZE], int rows, int cols) {
   int i, j;
   for(i = 0;i < rows; i++){
      for(j = 0;j < cols; j++){
         c[i][j] = a[i][j] + b[i][j];
      }
   }
}
```

i = 1, count = 9

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
              c[][MAX_SIZE], int rows, int cols) {
    int i, j;
    for(i = 0;i < rows; i++){
        for(j = 0;j < cols; j++){
            c[i][j] = a[i][j] + b[i][j];
        }
    }
}
```

j = 0, count = 10

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
              c[][MAX_SIZE], int rows, int cols) {
    int i, j;
    for(i = 0;i < rows; i++){
        for(j = 0;j < cols; j++){
            c[i][j] = a[i][j] + b[i][j];
        }
    }
}
```

count = 11

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
              c[][MAX_SIZE], int rows, int cols) {
    int i, j;
    for(i = 0;i < rows; i++){
        for(j = 0;j < cols; j++){
            c[i][j] = a[i][j] + b[i][j];
        }
    }
}
```

j = 1, count = 12

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
              c[][MAX_SIZE], int rows, int cols) {
    int i, j;
    for(i = 0;i < rows; i++){
        for(j = 0;j < cols; j++){
            c[i][j] = a[i][j] + b[i][j];
        }
    }
}
```

count = 13

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
            c[][MAX_SIZE], int rows, int cols) {
    int i, j;
    for(i = 0;i < rows; i++){
        for(j = 0;j < cols; j++){
            c[i][j] = a[i][j] + b[i][j];
        }
    }
}
```

j = 2, count = 14

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
            c[][MAX_SIZE], int rows, int cols) {
   int i, j;
   for(i = 0;i < rows; i++){
      for(j = 0;j < cols; j++){
         c[i][j] = a[i][j] + b[i][j];
      }
   }
}
```

count = 15

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
              c[][MAX_SIZE], int rows, int cols) {
    int i, j;
    for(i = 0;i < rows; i++){
        for(j = 0;j < cols; j++){
            c[i][j] = a[i][j] + b[i][j];
        }
    }
}
```

j = 3, count = 16

Program 1.17

initial
rows=2, cols=3, count = 0

i = 2, count = 17

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
          c[][MAX_SIZE], int rows, int cols) {
  int i, j;
  for(i = 0;i < rows; i++){
    for(j = 0;j < cols; j++){
      c[i][j] = a[i][j] + b[i][j];
    }
  }
}
```

回outline

Program 1.17

initial
rows=2, cols=3, count = 0

```
void add(int a[][MAX_SIZE], b[][MAX_SIZE],
               c[][MAX_SIZE], int rows, int cols) {
    int i, j;
    for(i = 0;i < rows; i++){
        for(j = 0;j < cols; j++){
            c[i][j] = a[i][j] + b[i][j];
        }
    }
}
```

rows+1 次

rows*(cols+1) 次

+ rows*cols 次

2rows*cols+2rows+1 次

Exercise 1

***Program 1.19:** Printing out a matrix

```
void print_matrix(int matrix[ ][MAX_SIZE], int rows, int cols)
{
   int i, j;
   for (i = 0; i < row; i++) {           /* row +1*/
      for (j = 0; j < cols; j++)          /*  row * (col +1) */
          printf("%d",  matrix[i][j]);   / row * col */
      printf( "\n");                        /* row */
   }
}
```

2*row*col + 2 row + row +1

Asymptotic Notation

Definition

- **Big-Oh (O)** *g(n)* is the upper bound of *f(n)*

  $f(n) = O(g(n))$ iff there exist positive constants c and n_0 such that $f(n) \leq cg(n)$, for all $n, n \geq n_0$.

- **Big-Omega (Ω)** *g(n)* is the lower bound of *f(n)*

  $f(n) = \Omega(g(n))$ iff there exist positive constants c and n_0 such that $f(n) \geq cg(n)$, for all $n, n \geq n_0$.

- **Big-Theta (Θ)**

  $f(n) = \Theta(g(n))$ iff there exist positive constants c_1, c_2 and n_0 such that $c_1 g(n) \leq f(n) \leq c_2 g(n)$, for all $n, n \geq n_0$.

Asymptotic Notation (O)

- Definition (Big (O))
  $f(n) = O(g(n))$ iff there exist positive constants c and n_0 such that $f(n) \le cg(n)$ for all n, $n \ge n_0$.
- Examples
  - $3n+2=O(n)$ /* $3n+2 \le 4n$ for $n \ge 2$ */
  - $3n+3=O(n)$ /* $3n+3 \le 4n$ for $n \ge 3$ */
  - $100n+6=O(n)$ /* $100n+6 \le 101n$ for $n \ge 6$ */
  - $10n^2+4n+2=O(n^2)$ /* $10n^2+4n+2 \le 11n^2$ for $n \ge 5$ */
  - $6*2^n+n^2=O(2^n)$ /* $6*2^n+n^2 \le 7*2^n$ for $n \ge 4$ */

Asymptotic Notation (Θ)

- Definition

  $f(n) = \Theta(g(n))$ iff there exist positive constants c_1, c_2 and n_0 such that $c_1 g(n) \le f(n) \le c_2 g(n)$, for all $n, n \ge n_0$.

- Examples
  - $3n + 2 = \Theta(n)$

    $3n + 2 \ge 3n$ for all $n \ge 2$ and $3n + 2 \le 4n$ for all $n \ge 2$,
    so $c_1 = 3, c_2 = 4$ and $n_0 = 2$
  - $10n^2 + 4n + 2 = \Theta(n^2)$

    $10n^2 + 4n + 2 \ge 10n^2$ for all $n \ge 5$ and $10n^2 + 4n + 2 \le 11n^2$ for all $n \ge 5$,
    so $c_1 = 10, c_2 = 11$ and $n_0 = 5$
  - $6 * 2^n + n^2 = \Theta(2^n)$

    $6 * 2^n + n^2 \ge 6 * 2^n$ for all $n \ge 4$ and $6 * 2^n + n^2 \le 7 * 2^n$ for all $n \ge 4$,
    so $c_1 = 6, c_2 = 7$ and $n_0 = 4$

Example

- Complexity of $c_1n^2+c_2n$ and c_3n
  - for sufficiently large of value, c_3n is faster than $c_1n^2+c_2n$
  - for small values of n, either could be faster
    - $c_1=1, c_2=2, c_3=100$ --> $c_1n^2+c_2n \le c_3n$ for $n \le 98$
    - $c_1=1, c_2=2, c_3=1000$ --> $c_1n^2+c_2n \le c_3n$ for $n \le 998$
  - break even point
    - no matter what the values of c1, c2, and c3, the n beyond which c_3n is always faster than $c_1n^2+c_2n$

- O(1): constant
- O(n): linear
- $O(n^2)$: quadratic
- $O(n^3)$: cubic
- $O(2^n)$: exponential
- O(logn)
- O(nlogn)

***Figure 1.7:**Function values

Instance characteristic n								
Time	Name	1	2	4	8	16	32
1	Constant	1	1	1	1	1	1
$\log n$	Logarithmic	0	1	2	3	4	5
n	Linear	1	2	4	8	16	32
$n \log n$	Log linear	0	2	8	24	64	160
n^2	Quadratic	1	4	16	64	256	1024
n^3	Cubic	1	8	64	512	4096	32768
2^n	Exponential	2	4	16	256	65536	4294967296
$n!$	Factorial	1	2	24	40326	20922789888000	26313×10^{33}

Figure 1.7 Function values

*Figure 1.8: Plot of function values

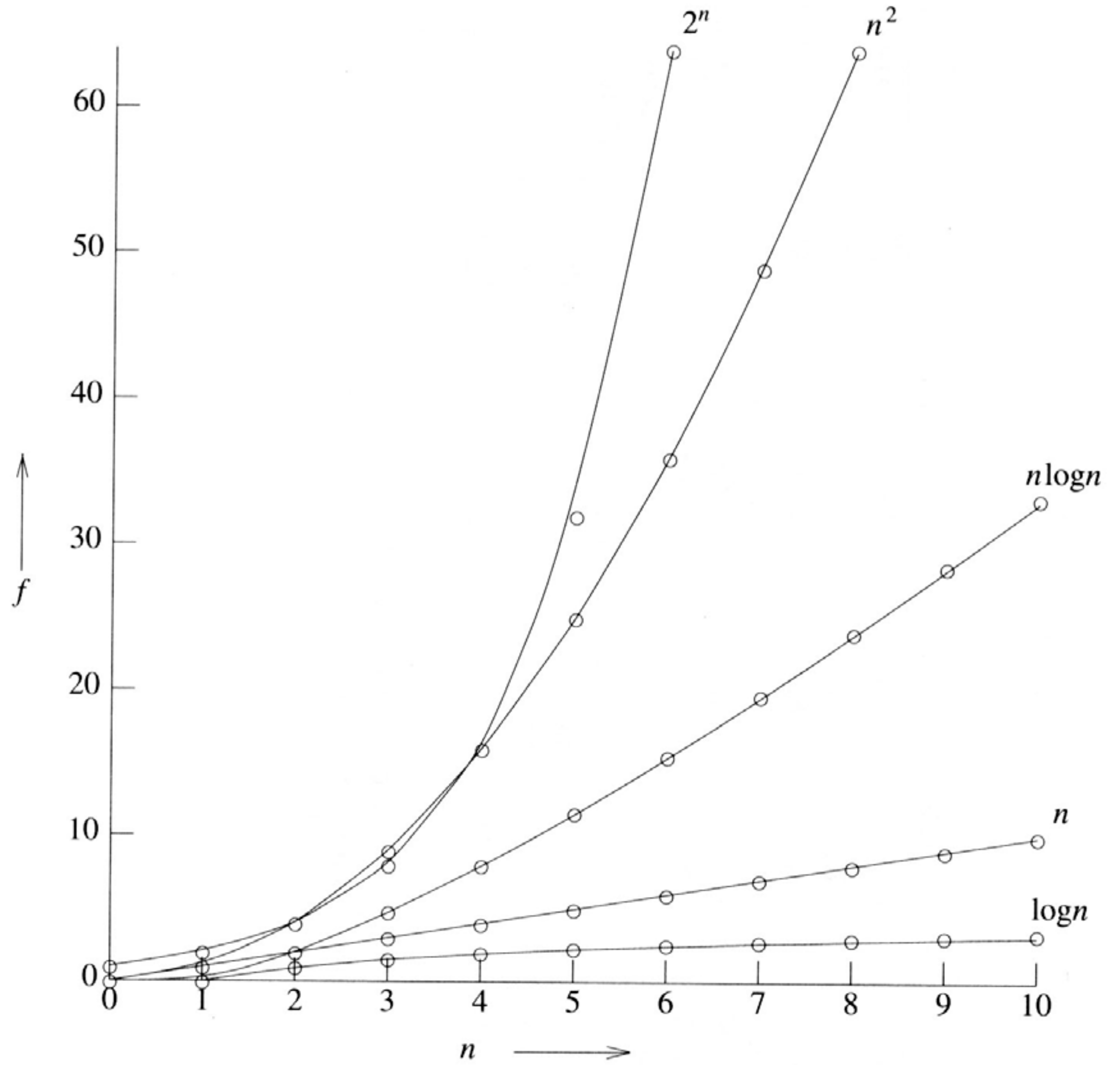

Figure 1.8 Plot of function values

***Figure 1.9:** Times on a 1 billion instruction per second computer

	$f(n)$						
n	n	$n \log_2 n$	n^2	n^3	n^4	n^{10}	2^n
10	.01 µs	.03 µs	.1 µs	1 µs	10 µs	10s	1µs
20	.02 µs	.09 µs	.4 µs	8 µs	160 µs	2.84h	1ms
30	.03 µs	.15 µs	.9 µs	27 µs	810 µs	6.83d	1s
40	.04 µs	.21 µs	1.6 µs	64 µs	2.56ms	121d	18m
50	.05 µs	.28 µs	2.5 µs	125 µs	6.25ms	3.1y	13d
100	.10 µs	.66 µs	10 µs	1ms	100ms	3171y	$4*10^{13}$y
10^3	1 µs	9.96 µs	1 ms	1s	16.67m	$3.17*10^{13}$y	$32*10^{283}$y
10^4	10 µs	130 µs	100 ms	16.67m	115.7d	$3.17*10^{23}$y	
10^5	100 µs	1.66 ms	10s	11.57d	3171y	$3.17*10^{33}$y	
10^6	1ms	19.92ms	16.67m	31.71y	$3.17*10^7$y	$3.17*10^{43}$y	