CHAPTER 3

STACKS AND QUEUES

All the programs in this file are selected from

Ellis Horowitz, Sartaj Sahni, and Susan Anderson-Freed
"Fundamentals of Data Structures in C",

Stack: a Last-In-First-Out (LIFO) list

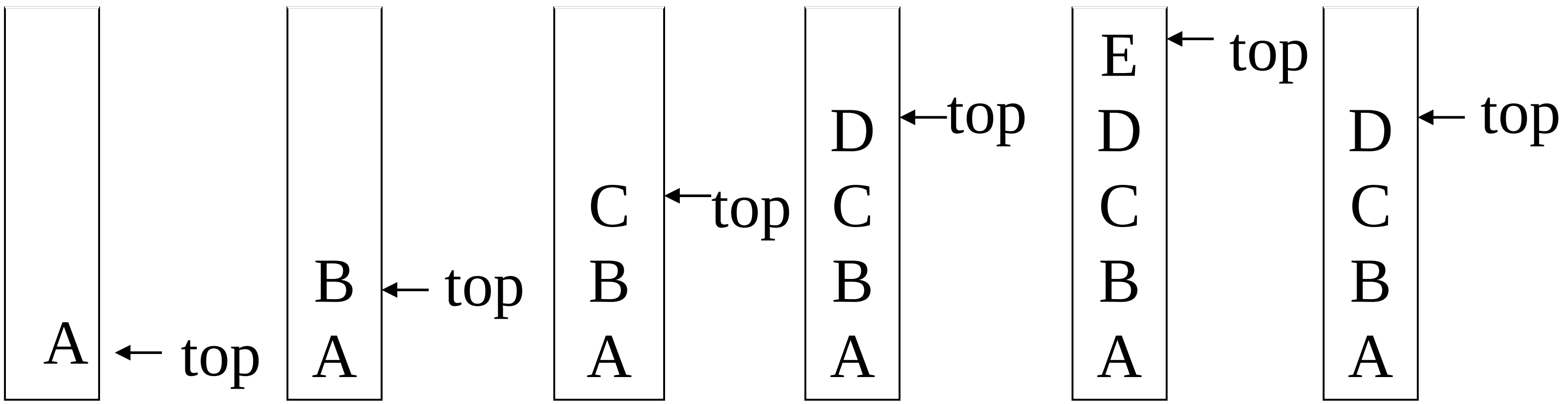


Figure 3.1: Inserting and deleting elements in a stack

An application of stack: stack frame of function call

(activation record)

fp: a pointer to current stack frame

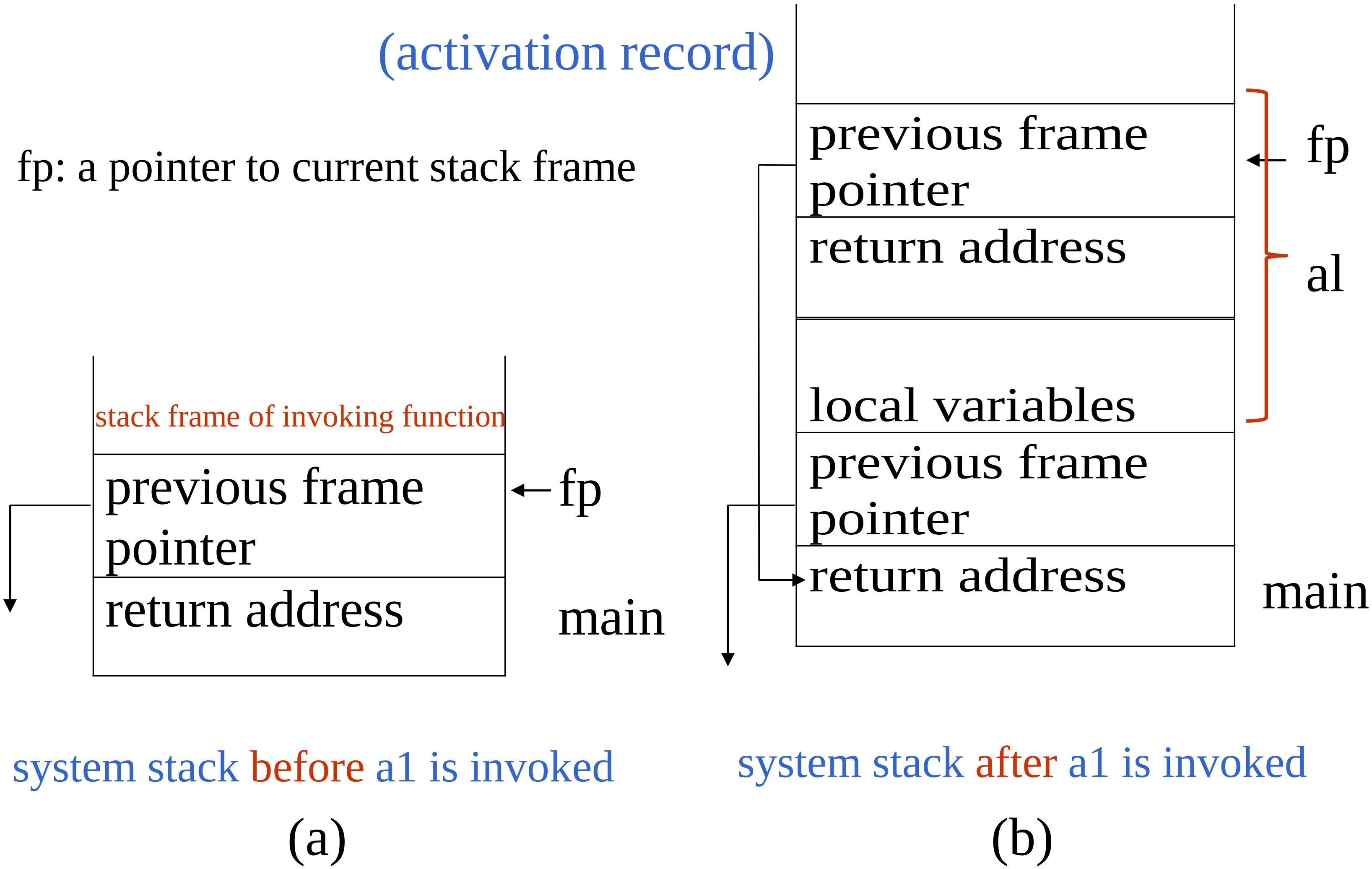


Figure 3.2: System stack after function call a1

abstract data type for stack

structure *Stack* is

objects: a finite ordered list with zero or more elements.

functions:

for all *stack* ∈ *Stack*, *item* ∈ *element*, *max_stack_size* ∈ positive integer

Stack CreateS(*max_stack_size*) ::=
    create an empty stack whose maximum size is *max_stack_size*

Boolean IsFull(*stack*, *max_stack_size*) ::=
    if (number of elements in *stack* == *max_stack_size*)
    return TRUE
    else return FALSE

Stack Push(*stack*, *item*) ::=
    if (IsFull(*stack*)) **return** *stack_full*
    else insert *item* into top of *stack*

Boolean IsEmpty(*stack*) ::=
if(*stack* == CreateS(*max_stack_size*))
return TRUE
else return FALSE
Element Pop(*stack*) ::=
if(IsEmpty(*stack*)) **return** *stack_empty*
else remove and return the *item* on the top of the stack.

Structure 3.1: Abstract data type *Stack*

Implementation: using array

***Stack* CreateS(max_stack_size)** ::=

```
#define MAX_STACK_SIZE 100 /* maximum stack size */
typedef struct {
            int key;
            /* other fields */
            } element;
element stack[MAX_STACK_SIZE];
int top = -1;
```

***Boolean* IsEmpty(Stack)** ::= top< 0;

***Boolean* IsFull(Stack)** ::= top >= MAX_STACK_SIZE-1;

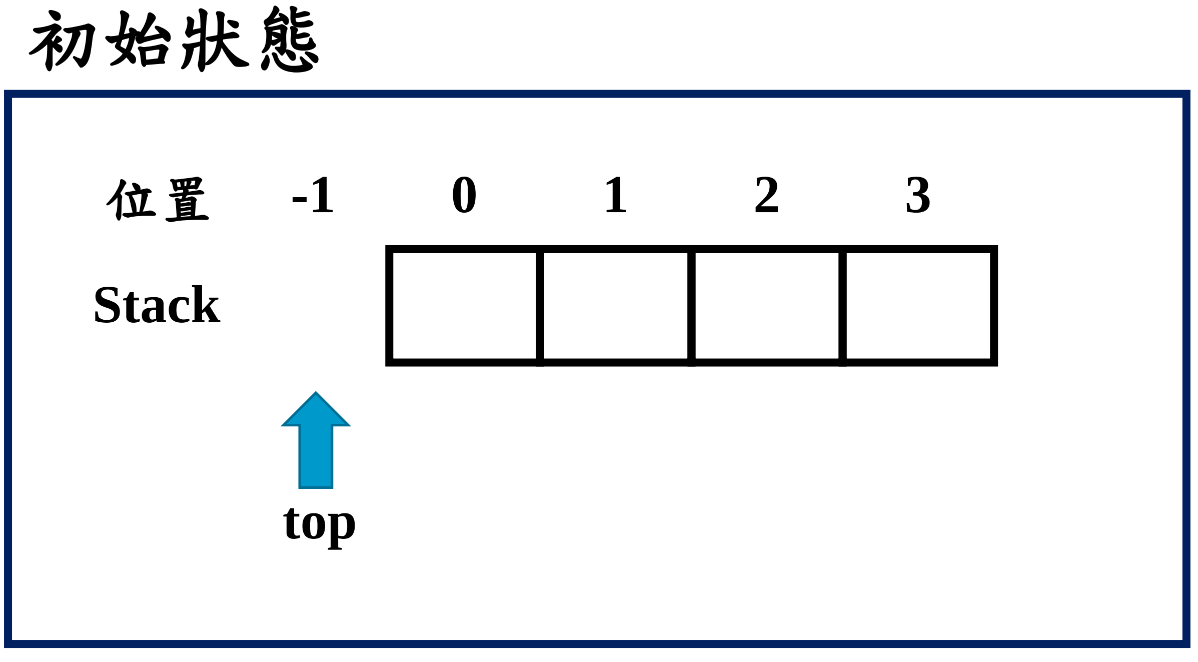
Implementation stack by array

- 用array實作出stack的新增和刪除功能
- 主要三個function
  1. 插入: push
  2. 刪除: pop
  3. Stack是否為空的: isEmpty

Implementation stack by array

– 插入: push

依序插入 10, 25, 14

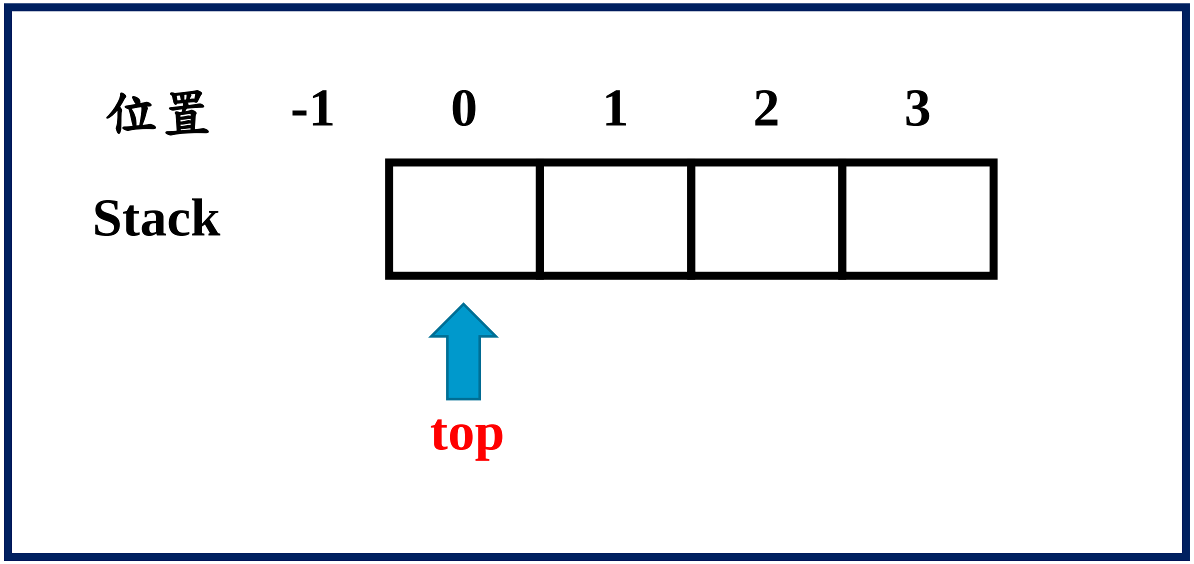


Implementation stack by array

– 插入: push

依序插入 10, 25, 14

top+1

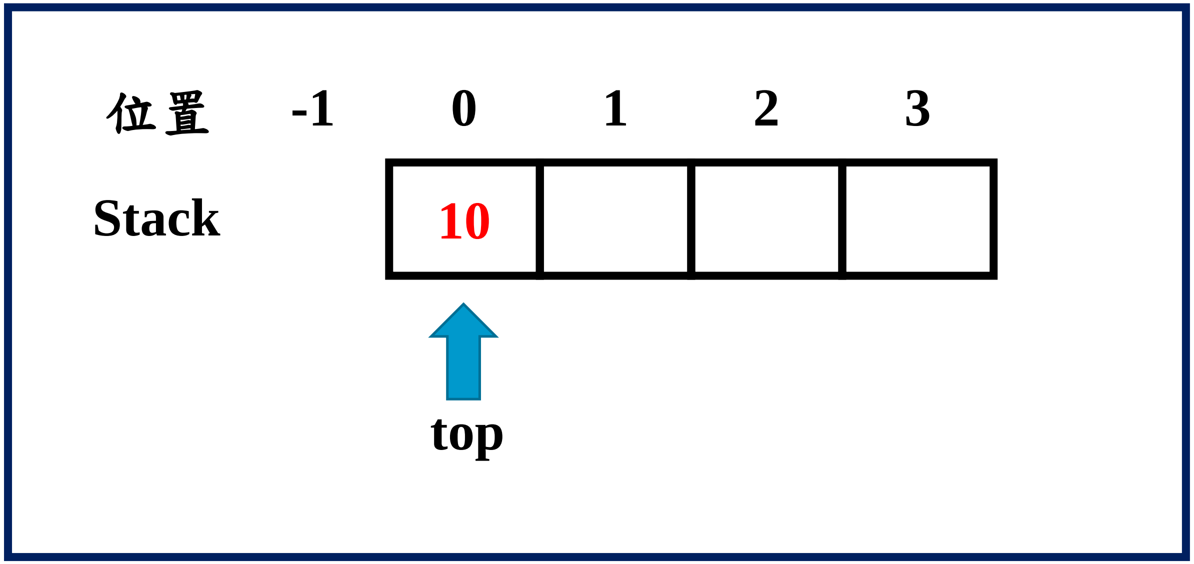


Implementation stack by array

– 插入: push

依序插入 10, 25, 14

插入10

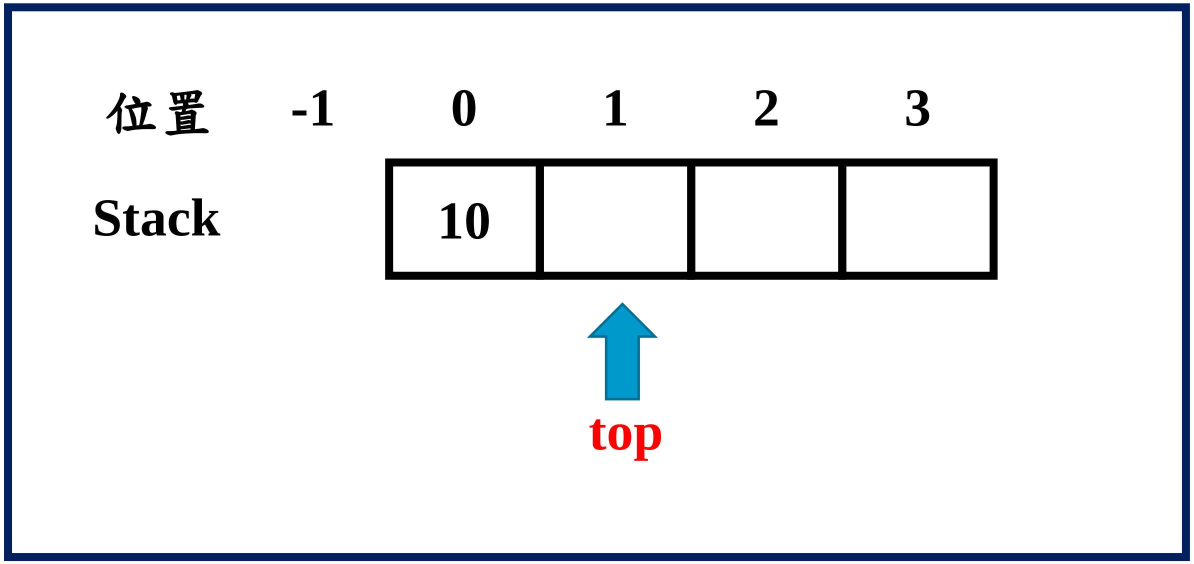


Implementation stack by array

– 插入: push

依序插入 10, 25, 14

top+1

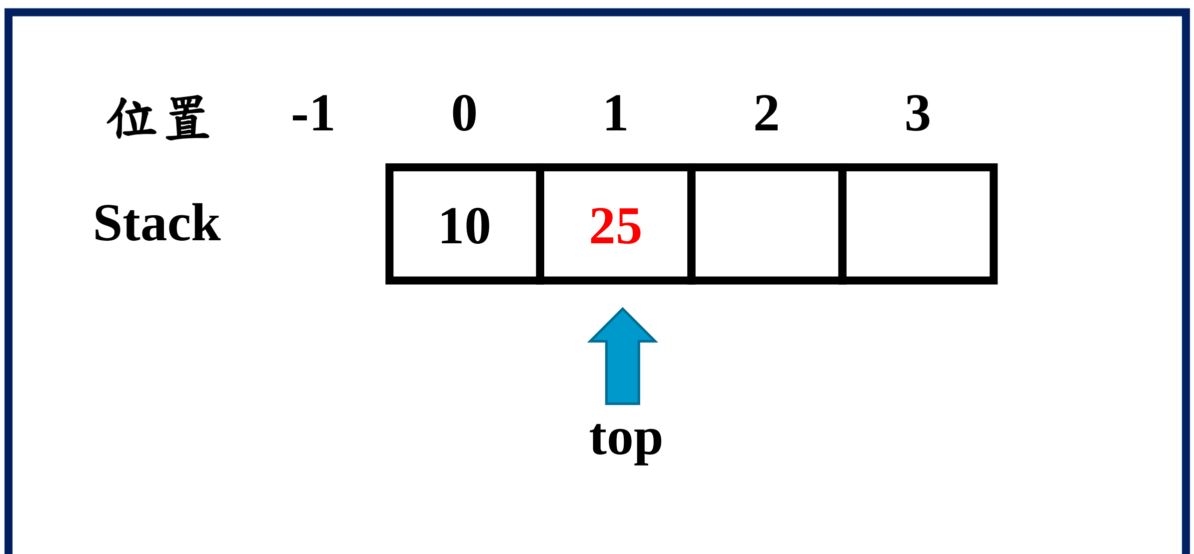


Implementation stack by array

– 插入: push

依序插入 10, 25, 14

插入25

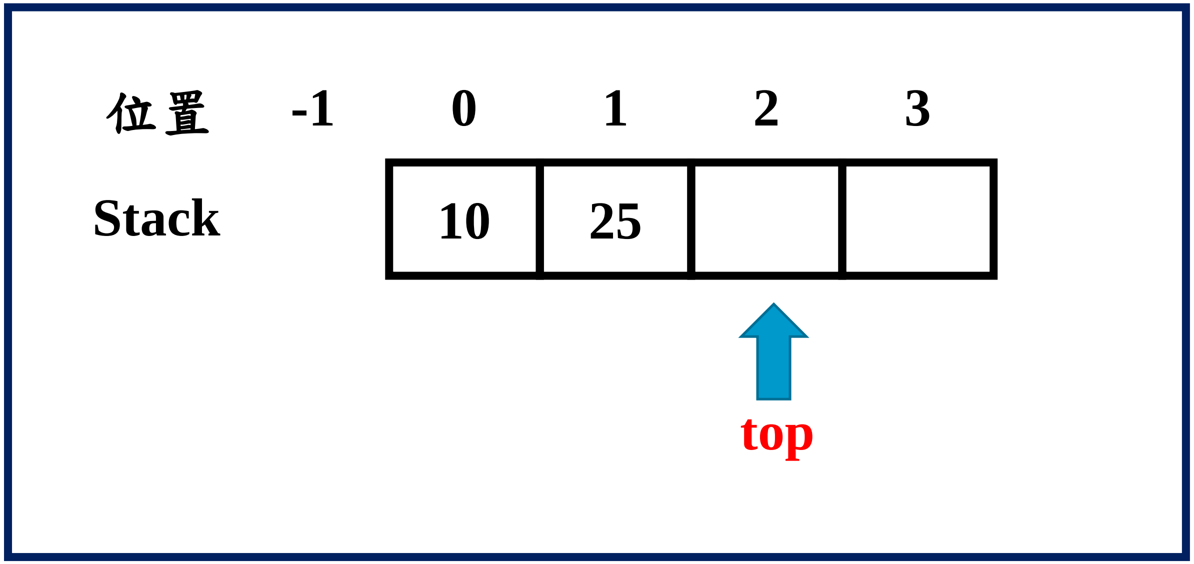


Implementation stack by array

– 插入: push

依序插入 10, 25, 14

top+1

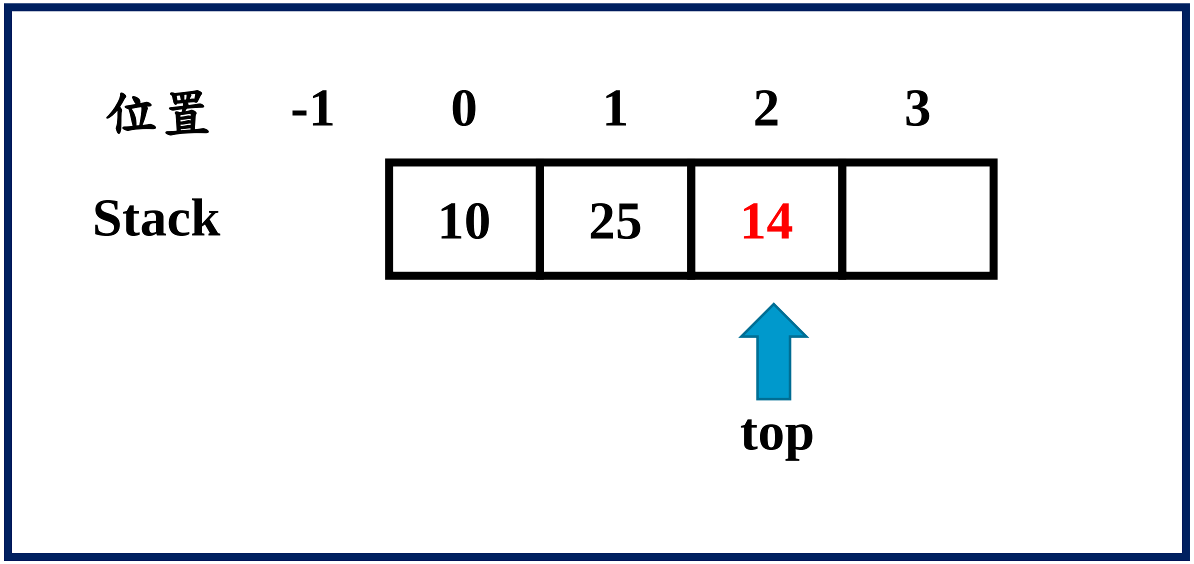


Implementation stack by array

– 插入: push

依序插入 10, 25, 14

插入14



Implementation stack by array

– 插入: push

```
void push(element item){
    if (top >= MAX_STACK_SIZE-1)
        stackFull();
    stack[++top] = item;
}
```

超出array大小時無法插入資料

將top移到要插入資料的位置並插入資料

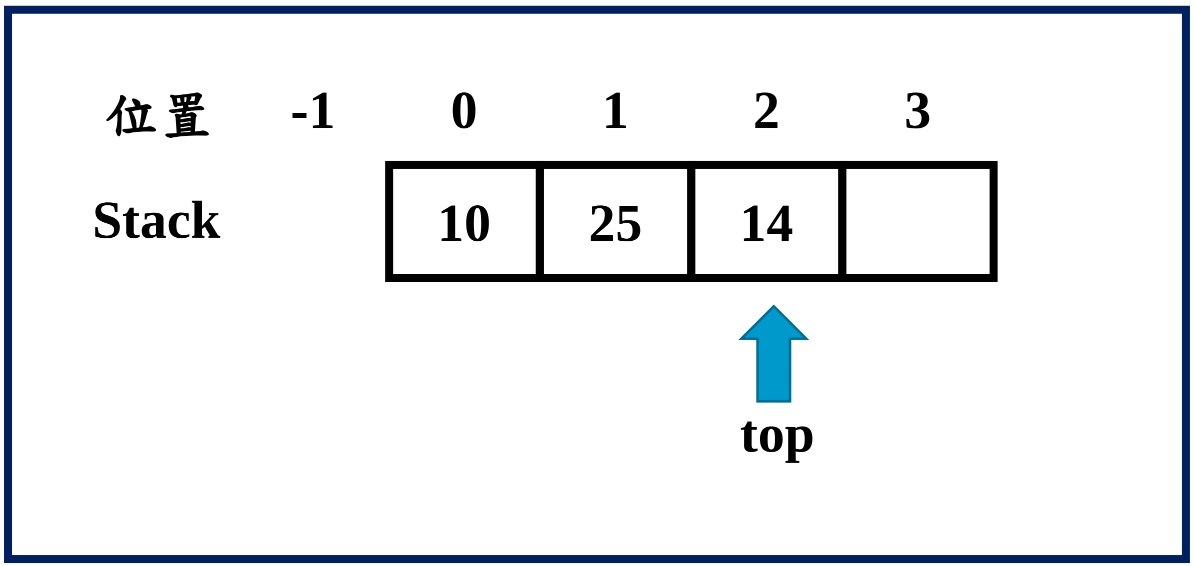
```
void stackFull(){
    fprintf(stderr, "Stack is full, cannot add element\n");
    exit(EXIT_FAILURE);
}
```

Implementation stack by array

– 刪除: pop

刪除順序依照”後進先出”

目前stack

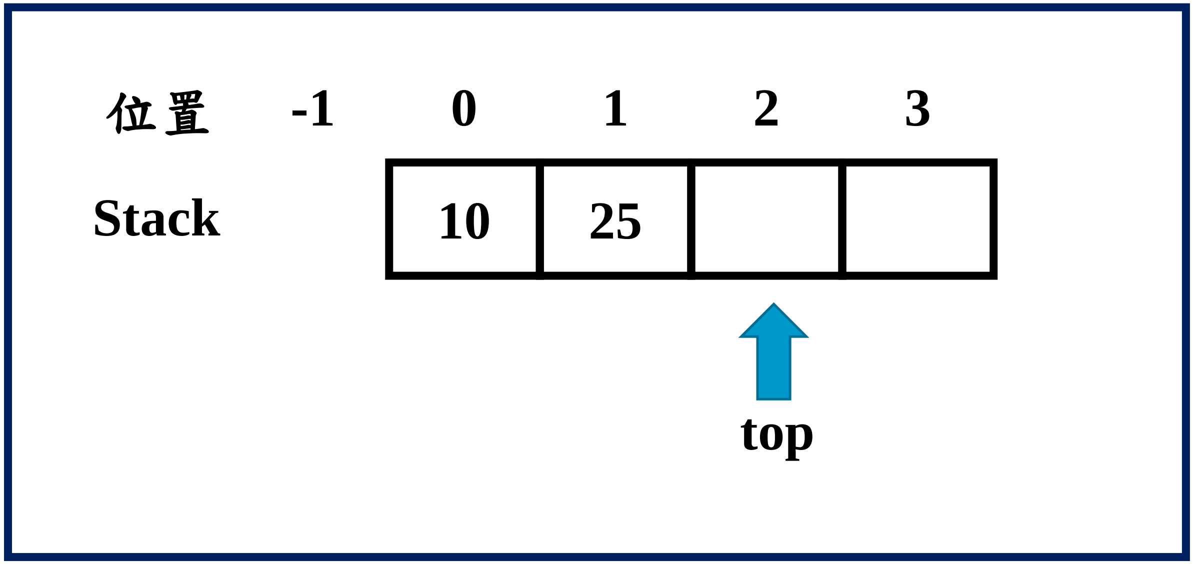


Implementation stack by array

– 刪除: pop

刪除順序依照”後進先出”

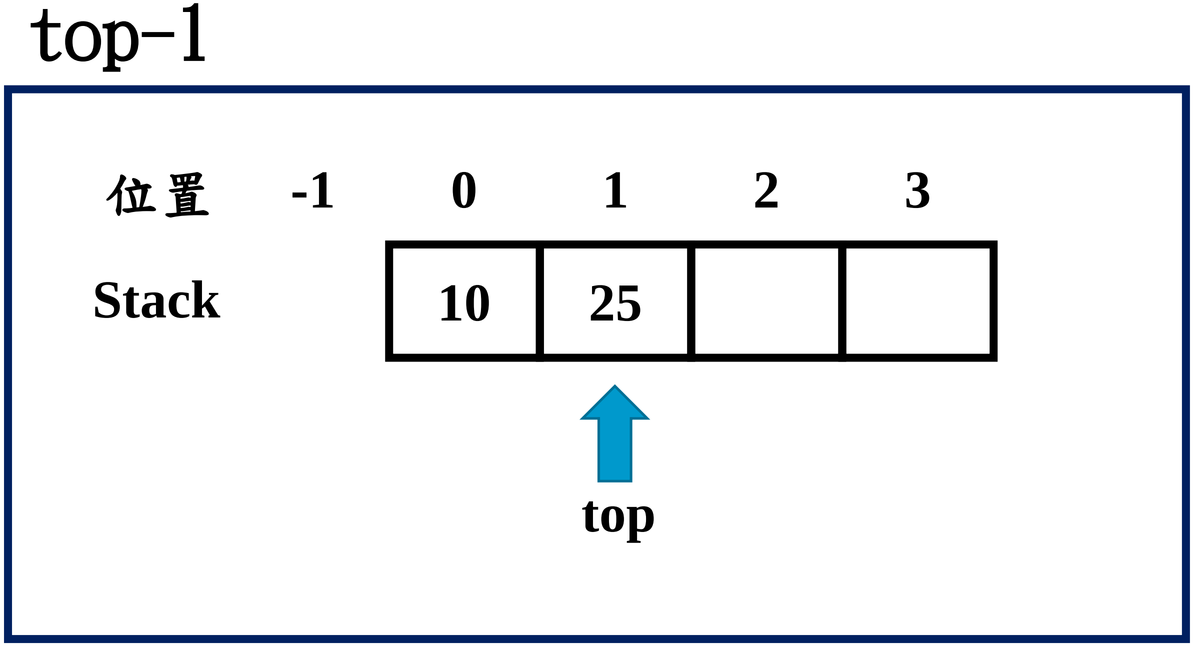
刪除14



Implementation stack by array

– 刪除: pop

刪除順序依照”後進先出”

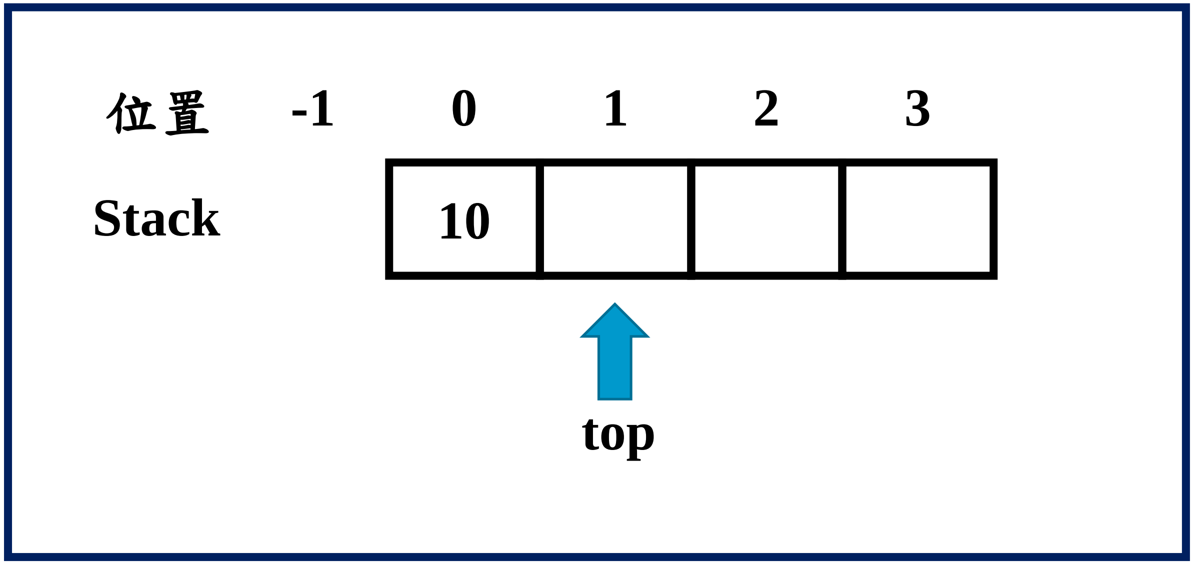


Implementation stack by array

– 刪除: pop

刪除順序依照”後進先出”

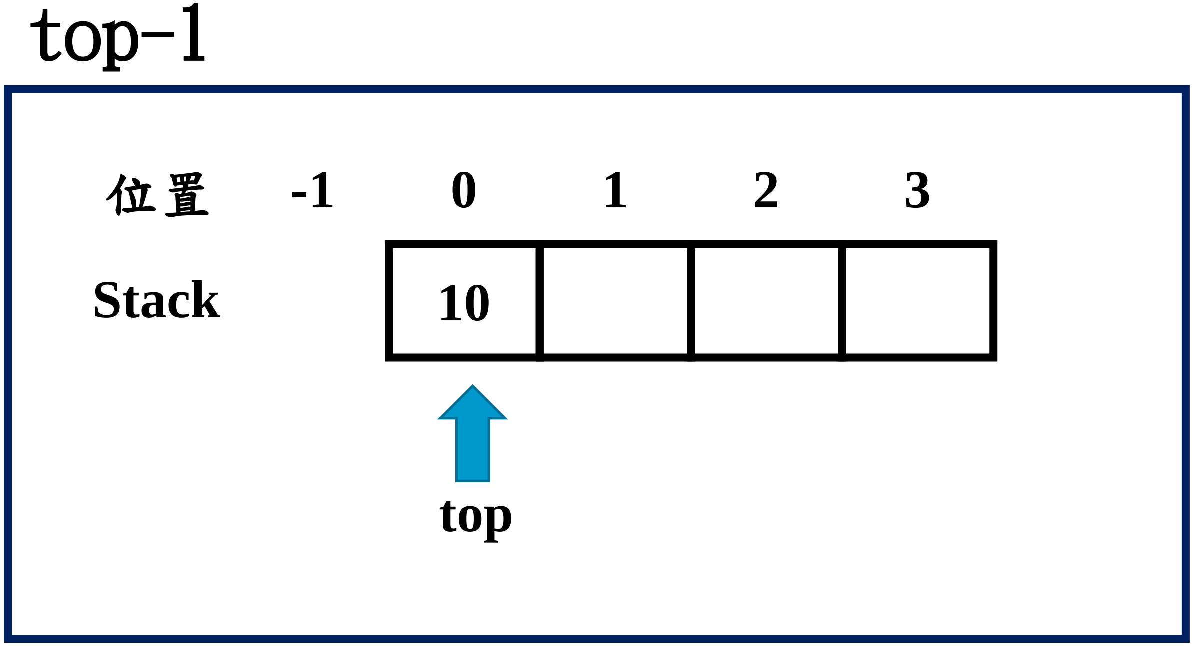
刪除25



Implementation stack by array

– 刪除: pop

刪除順序依照"後進先出"

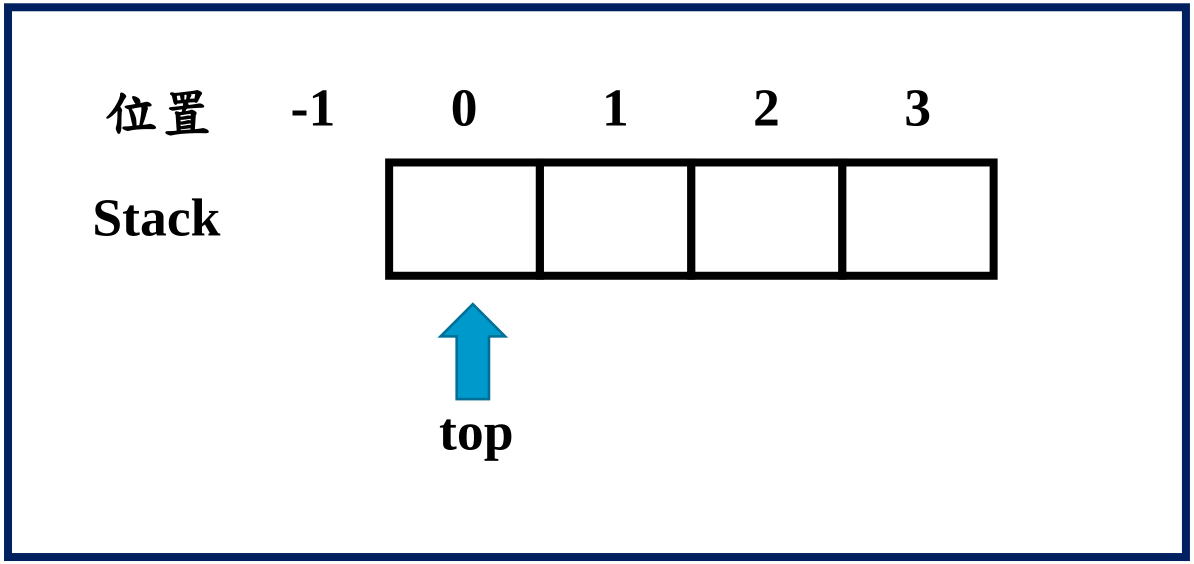


Implementation stack by array

– 刪除: pop

刪除順序依照”後進先出”

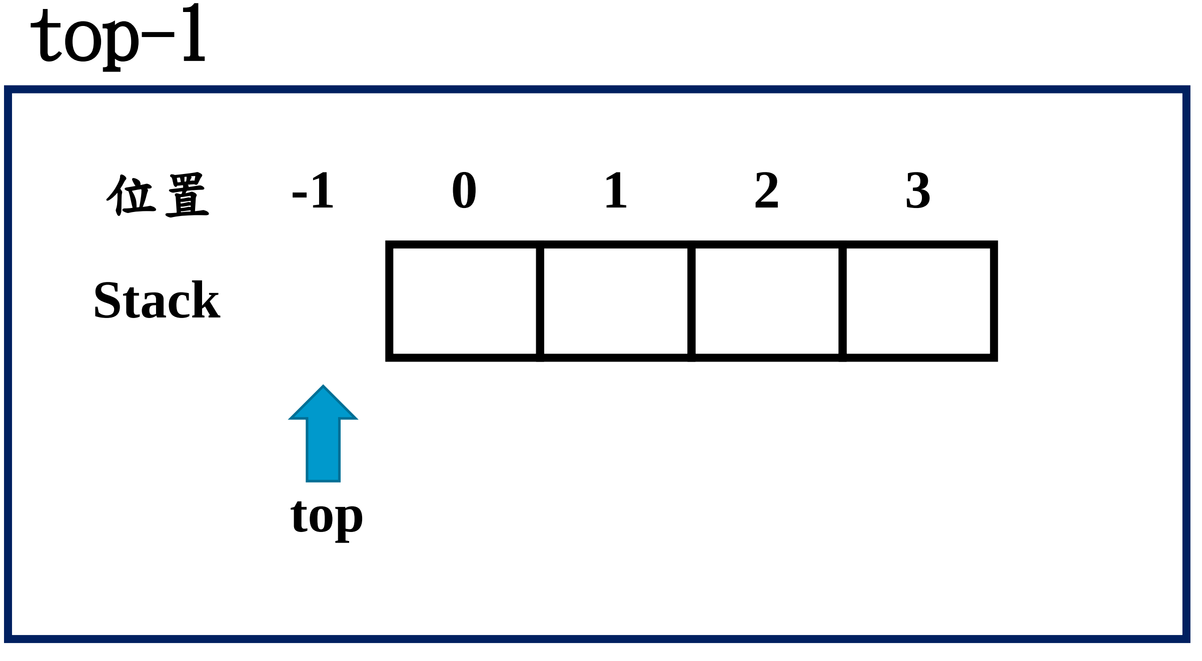
刪除10



Implementation stack by array

– 刪除: pop

刪除順序依照”後進先出”



Implementation stack by array

– 刪除: pop

```
element pop(){
    if (top == -1)
        return stackEmpty();
    return stack[top--];
}
```

array是空的則無法刪除資料

指到的資料後回傳

```
element stackEmpty(){
    fprintf(stderr, "Stack is empty, cannot delete element\n");
    return errorKey;
}
```

Queue: a First-In-First-Out (FIFO) list

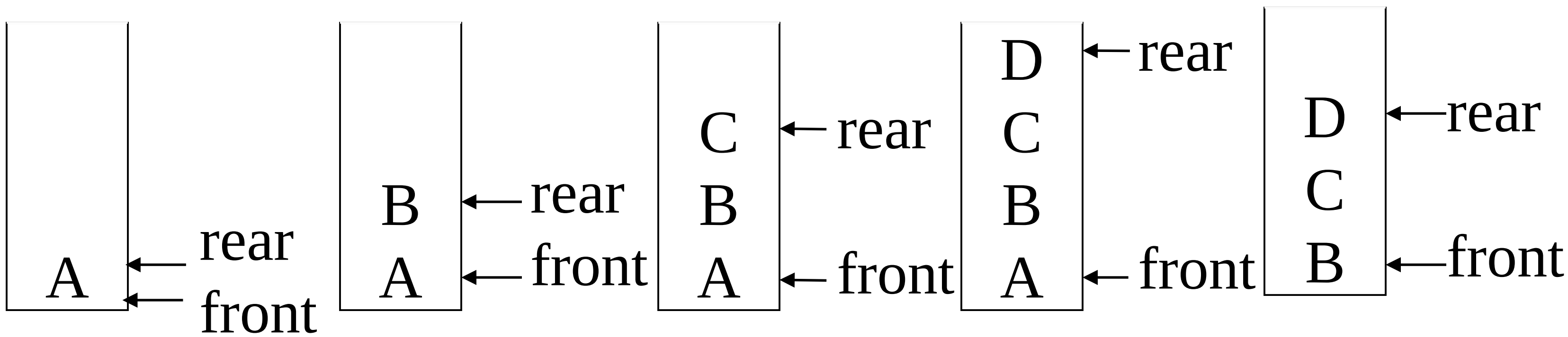


Figure 3.4: Inserting and deleting elements in a queue

Application: Job scheduling

front	rear	Q[0]	Q[1]	Q[2]	Q[3]	Comments
-1	-1					queue is empty
-1	0	J1				Job 1 is added
-1	1	J1	J2			Job 2 is added
-1	2	J1	J2	J3		Job 3 is added
0	2		J2	J3		Job 1 is deleted
1	2			J3		Job 2 is deleted

Figure 3.5: Insertion and deletion from a sequential queue

Abstract data type of queue

structure *Queue* is
objects: a finite ordered list with zero or more elements.
functions:
for all *queue* ∈ *Queue*, *item* ∈ *element*,
max_ queue_ size ∈ positive integer
Queue CreateQ(*max_queue_size*) ::=
create an empty queue whose maximum size is
max_queue_size
Boolean IsFullQ(*queue*, *max_queue_size*) ::=
if(number of elements in *queue* == *max_queue_size*)
return *TRUE*
else return *FALSE*
Queue AddQ(*queue*, *item*) ::=
if (IsFullQ(*queue*)) *queue_full*
else insert *item* at rear of *queue* and return *queue*

Boolean IsEmptyQ(*queue*) ::=
    if (*queue* ==CreateQ(*max_queue_size*))
    return *TRUE*
    else return *FALSE*
Element DeleteQ(*queue*) ::=
    if (IsEmptyQ(*queue*)) **return**
    else remove and return the *item* at front of queue.

Structure 3.2: Abstract data type *Queue*

Implementation 1: using array

```
Queue CreateQ(max_queue_size) ::=
# define MAX_QUEUE_SIZE 100/* Maximum queue size */
typedef struct {
                int key;
                /* other fields */
                } element;
element queue[MAX_QUEUE_SIZE];
int rear = -1;
int front = -1;
Boolean IsEmpty(queue) ::= front == rear
Boolean IsFullQ(queue) ::= rear == MAX_QUEUE_SIZE-1
```

Implementation queue by array

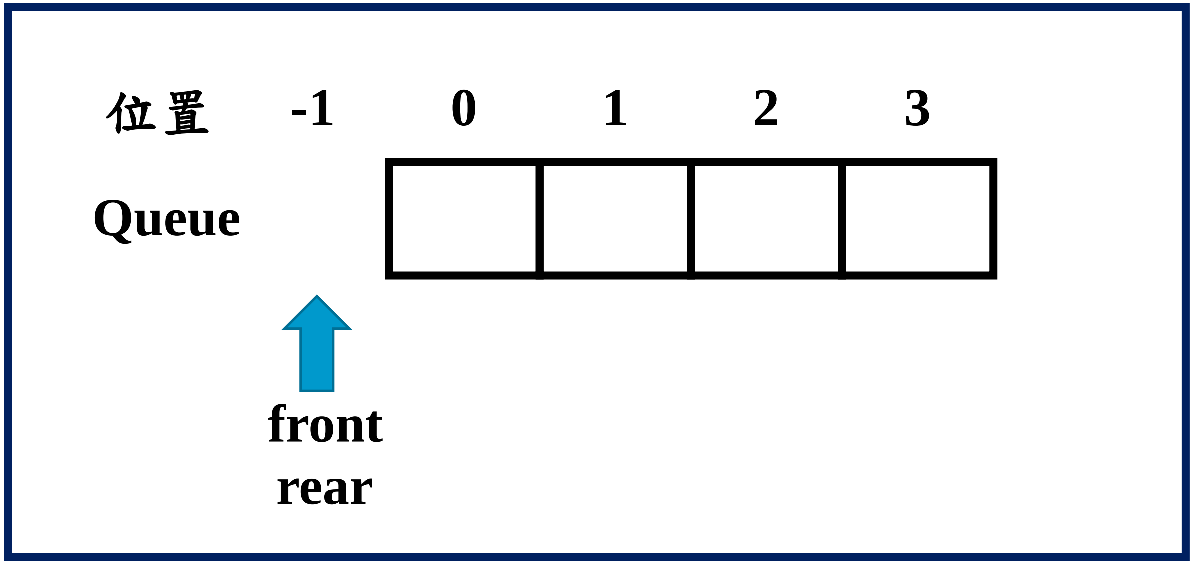
- 用array實作出queue的新增和刪除功能
- 主要三個function
  1. 插入: addq
  2. 刪除: deleteq
  3. Queue是否為空的: isEmpty

Implementation queue by array

– 插入: addq

依序插入 10, 25, 14

初始狀態

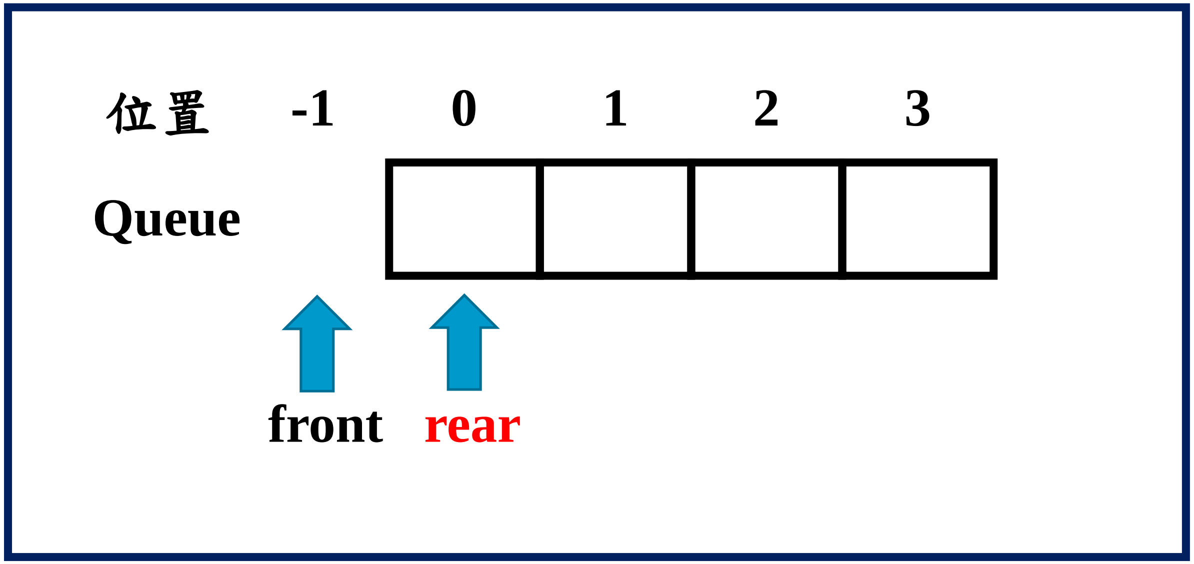


Implementation queue by array

– 插入: addq

依序插入 10, 25, 14

rear+1

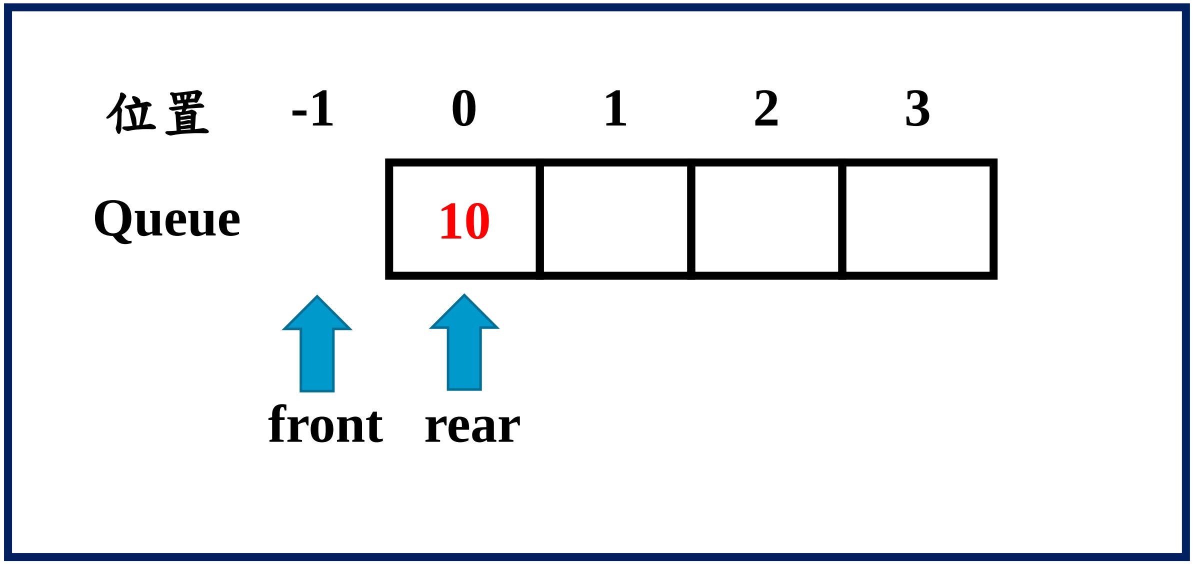


Implementation queue by array

– 插入: addq

依序插入 10, 25, 14

插入10

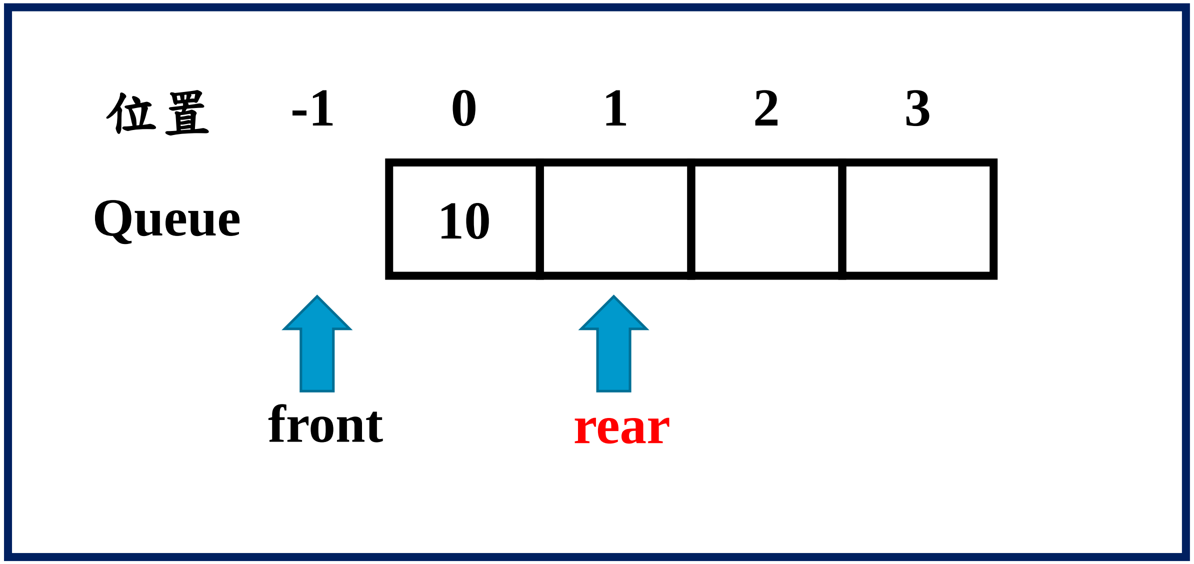


Implementation queue by array

– 插入: addq

依序插入 10, 25, 14

rear+1

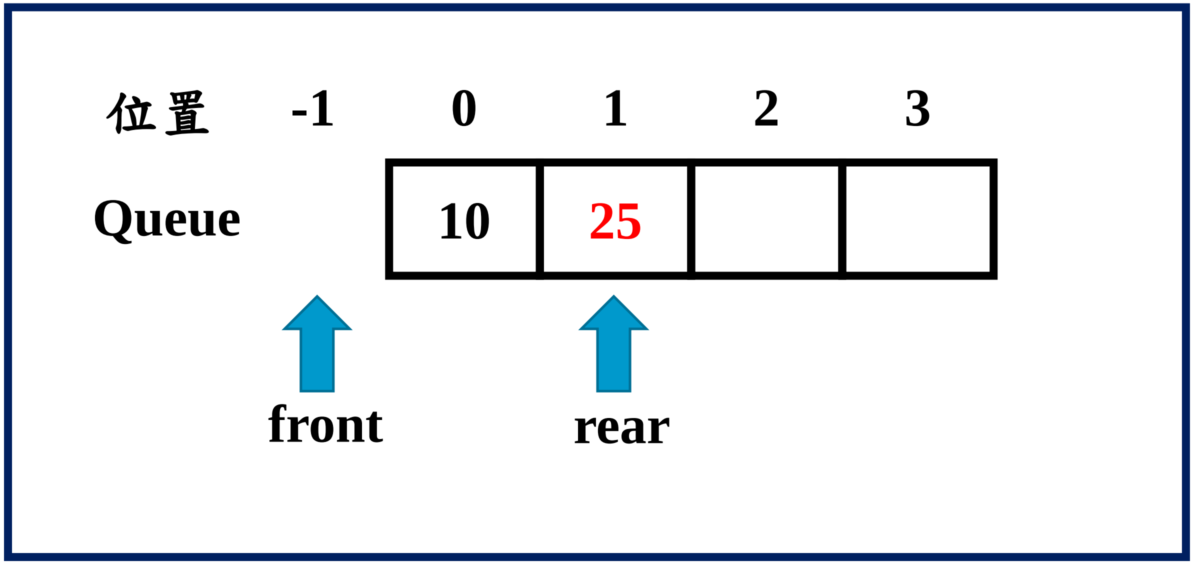


Implementation queue by array

– 插入: addq

依序插入 10, 25, 14

插入25

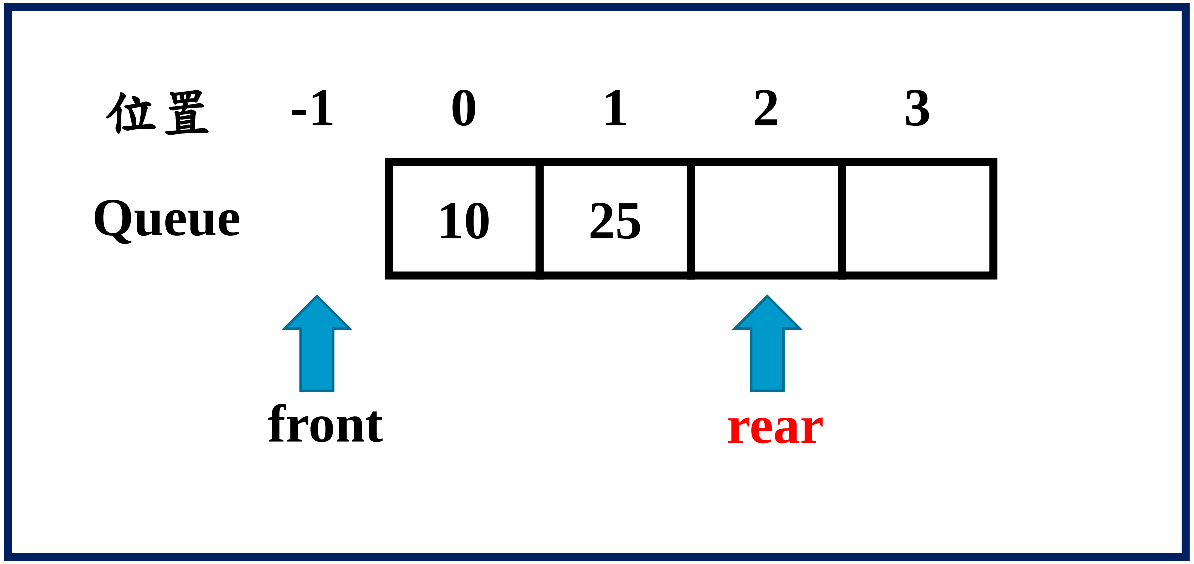


Implementation queue by array

– 插入: addq

依序插入 10, 25, 14

rear+1

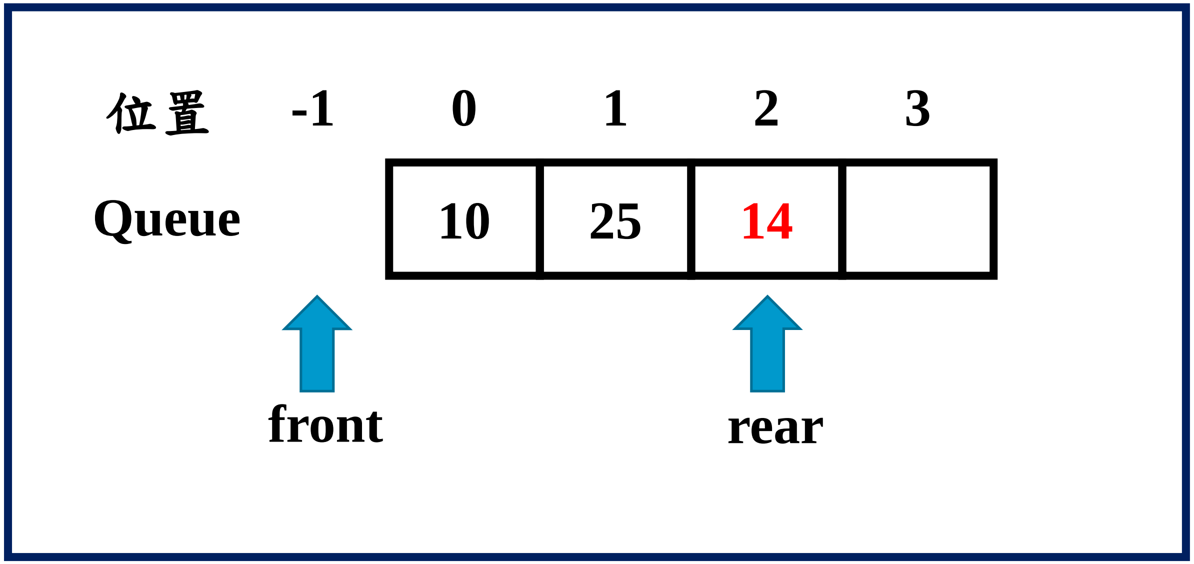


Implementation queue by array

– 插入: addq

依序插入 10, 25, 14

插入14



Implementation queue by array

– 插入: addq

```
void addq(element item){
    if (rear == MAX_QUEUE_SIZE-1)
        queueFull();
    queue[++rear] = item;
}
```

array是滿的則無法插入資料

將rear移到要增加資料的位置並插入資料

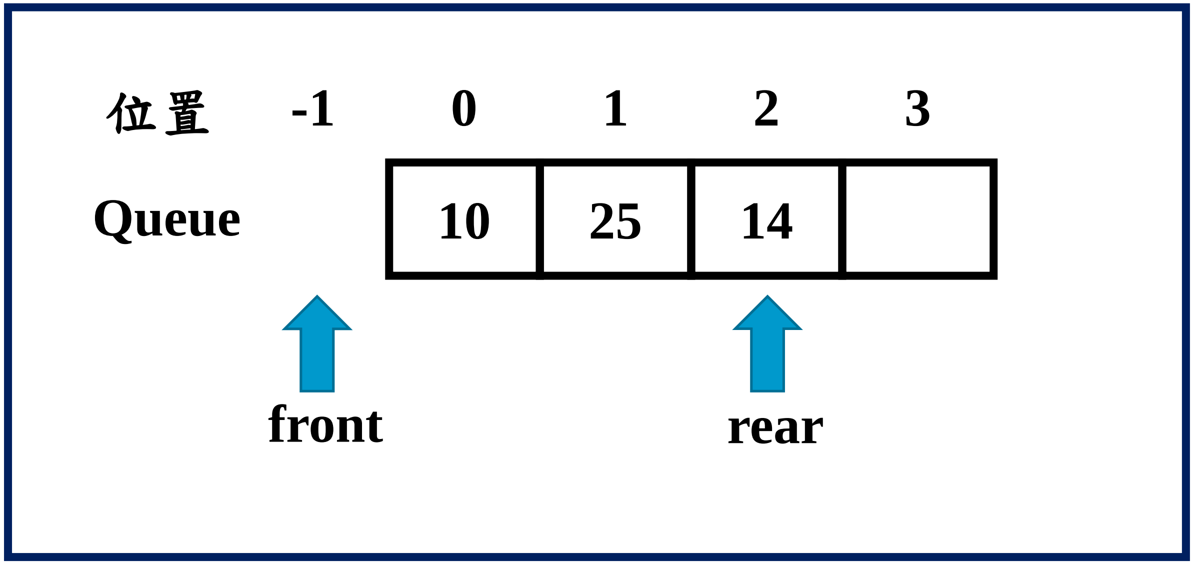
```
void queueFull(){
    fprintf(stderr, "Queue is full, cannot add element\n");
    exit(EXIT_FAILURE);
}
```

Implementation queue by array

– 刪除: deleteq

刪除順序依照”先進先出”

目前queue

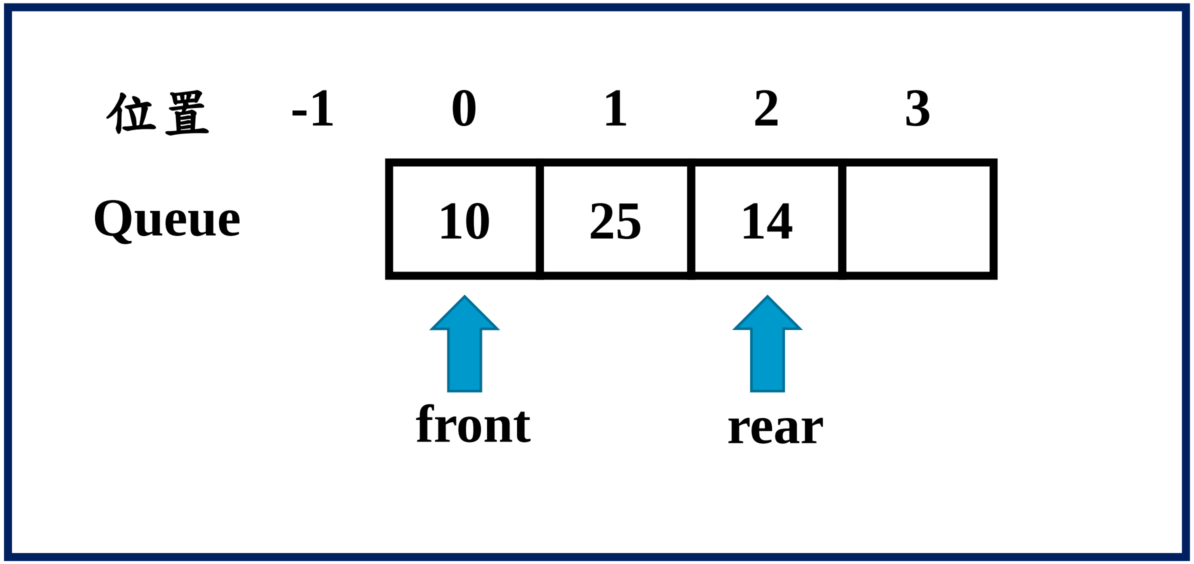


Implementation queue by array

– 刪除: deleteq

刪除順序依照”先進先出”

front+1

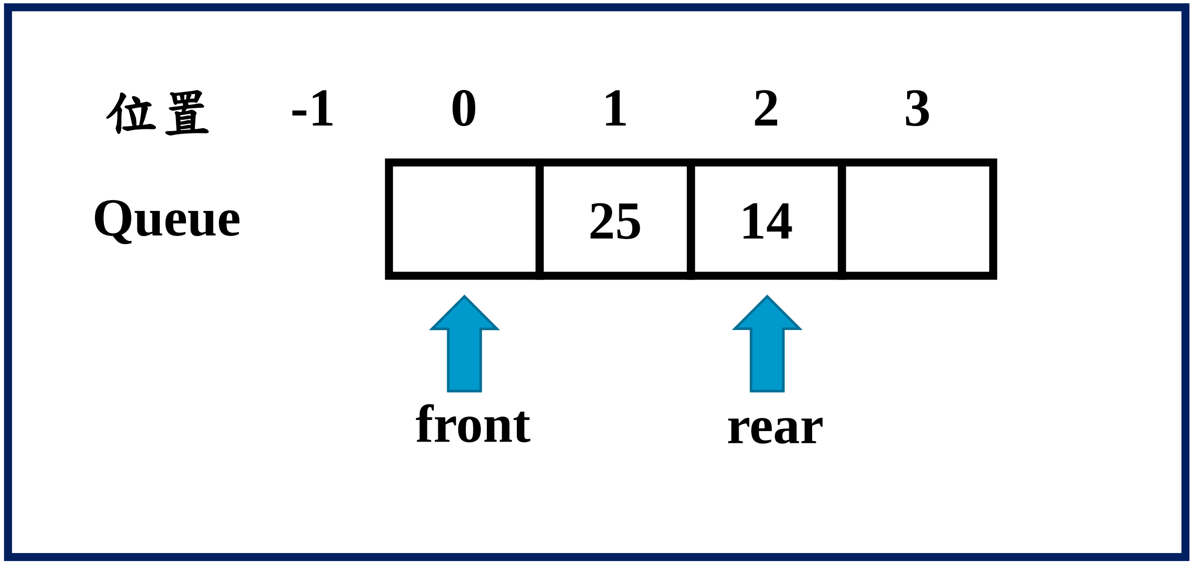


Implementation queue by array

– 刪除: deleteq

刪除順序依照”先進先出”

刪除10

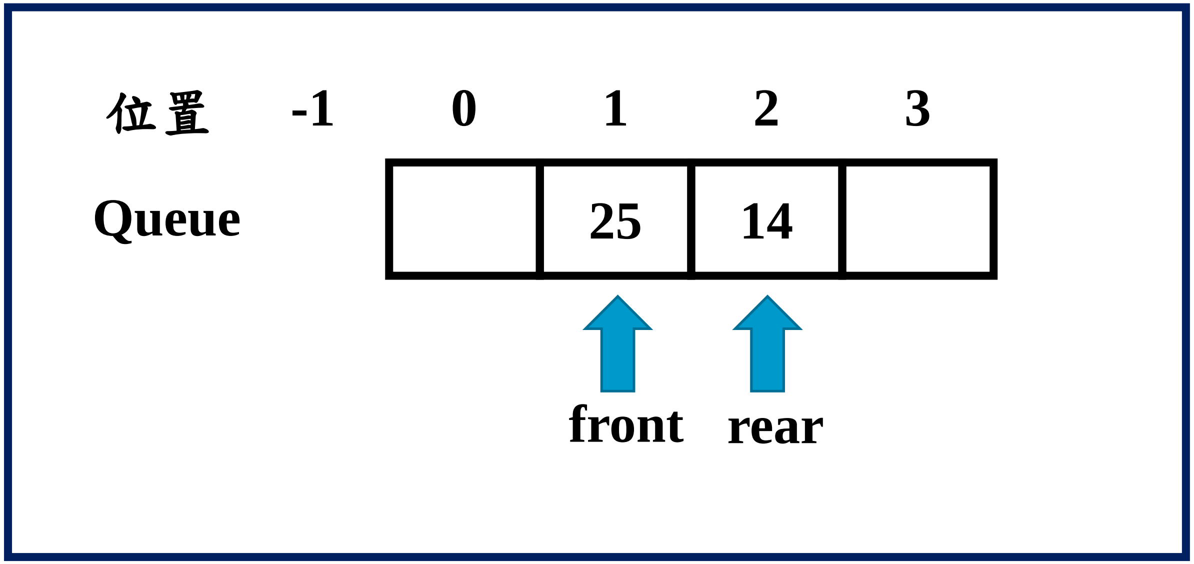


Implementation queue by array

– 刪除: deleteq

刪除順序依照”先進先出”

front+1

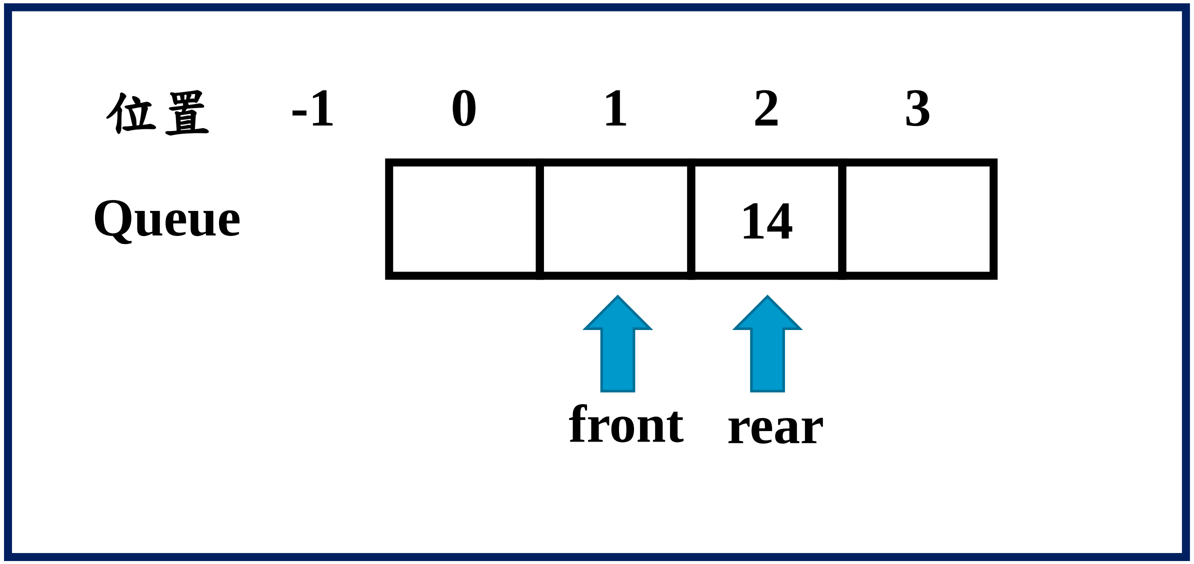


Implementation queue by array

– 刪除: deleteq

刪除順序依照”先進先出”

刪除25

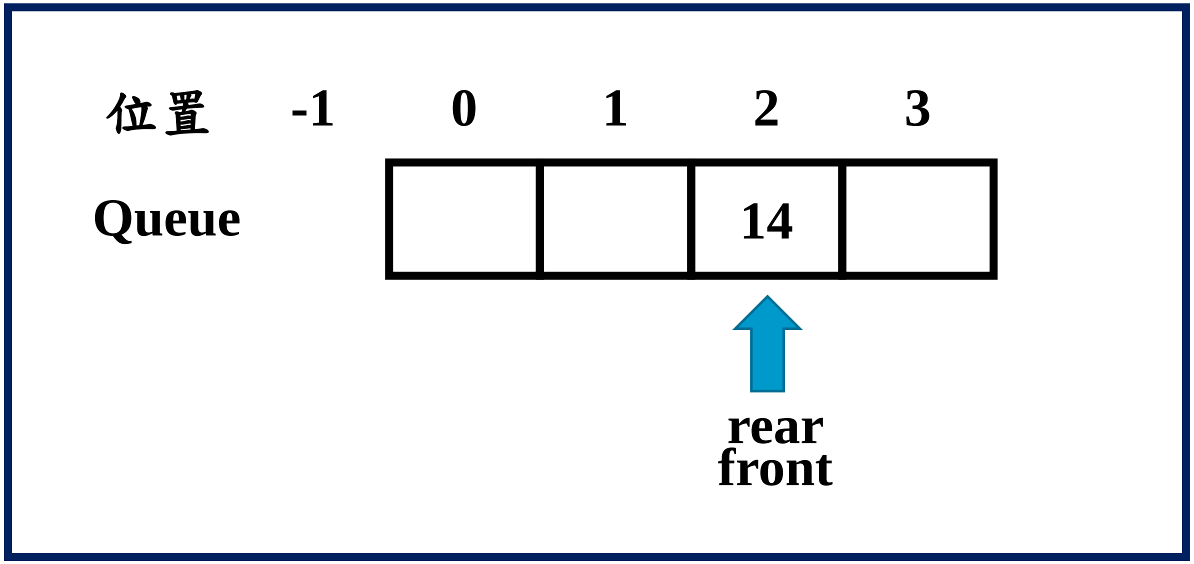


Implementation queue by array

– 刪除: deleteq

刪除順序依照"先進先出"

front+1

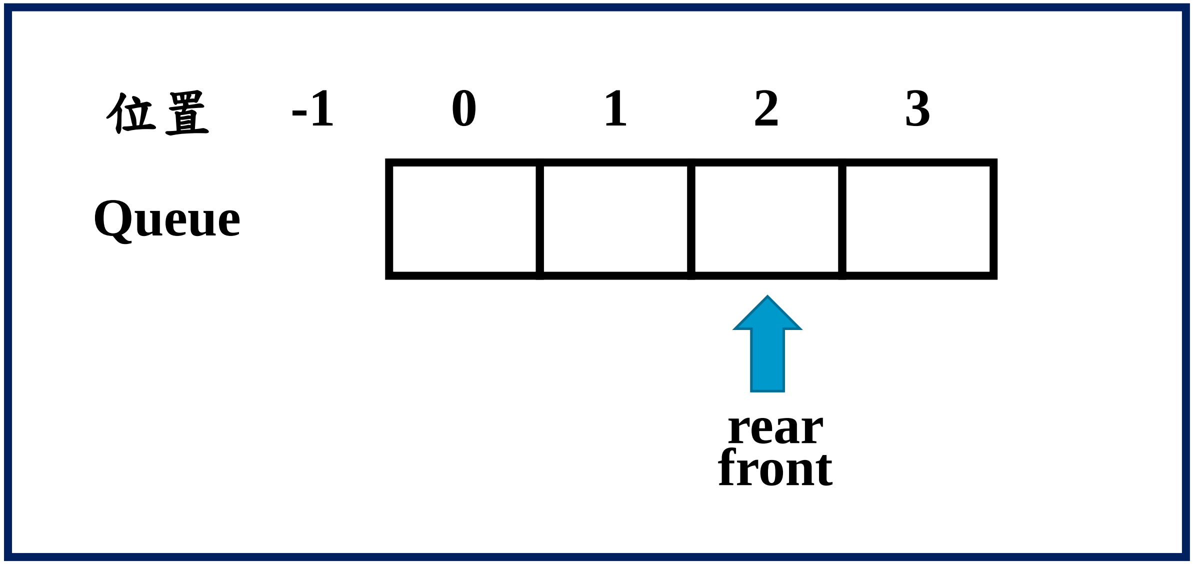


Implementation queue by array

– 刪除: deleteq

刪除順序依照”先進先出”

刪除14



Implementation queue by array

– 刪除: deleteq

```
element deleteq(){
    if (front == rear)
        return queueEmpty();
    return queue[++front];
}
```

array是空的則無法刪除資料

將front往要刪除的資料方向移動並刪除資料

```
element queueEmpty(){
    fprintf(stderr, "Queue is empty, cannot delete element\n");
    return errorKey;
}
```

Delete from a queue

```
element deleteq(int front, int rear)
{
/* remove element at the front of the queue */
    if ( front == rear)
        return queue_empty( );     /* return an error key */
    return queue [++ front];
}
```

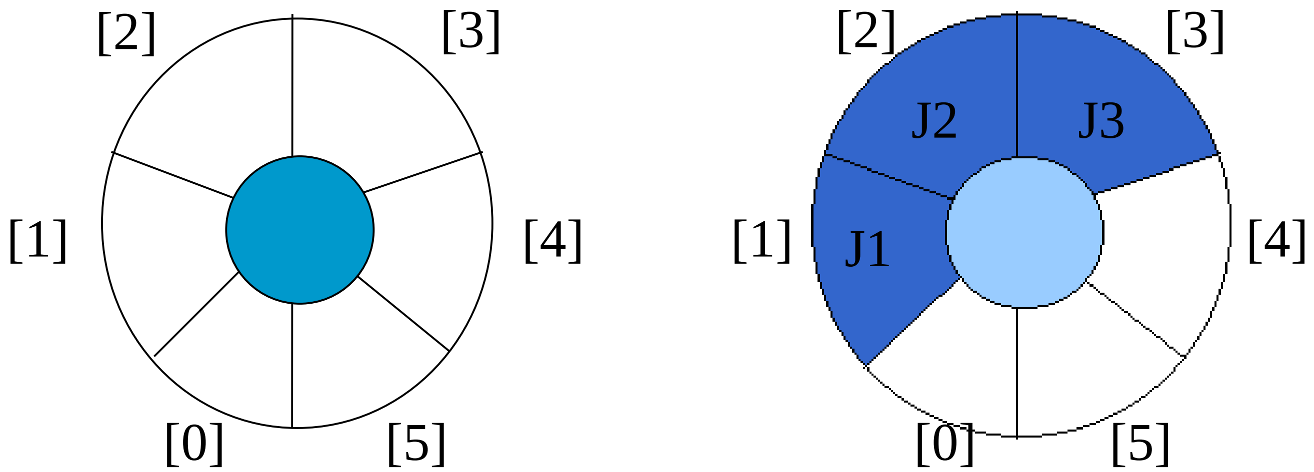
Program 3.6: Delete from a queue

problem: there may be available space when IsFullQ is true i.e., movement is required.

Implementation 2: regard an array as a circular queue

front: one position counterclockwise from the first element
rear: current end

EMPTY QUEUE



front = 0
rear = 0

front = 0
rear = 3

Figure 3.6: Empty and nonempty circular queues

Problem: one space is left when queue is full

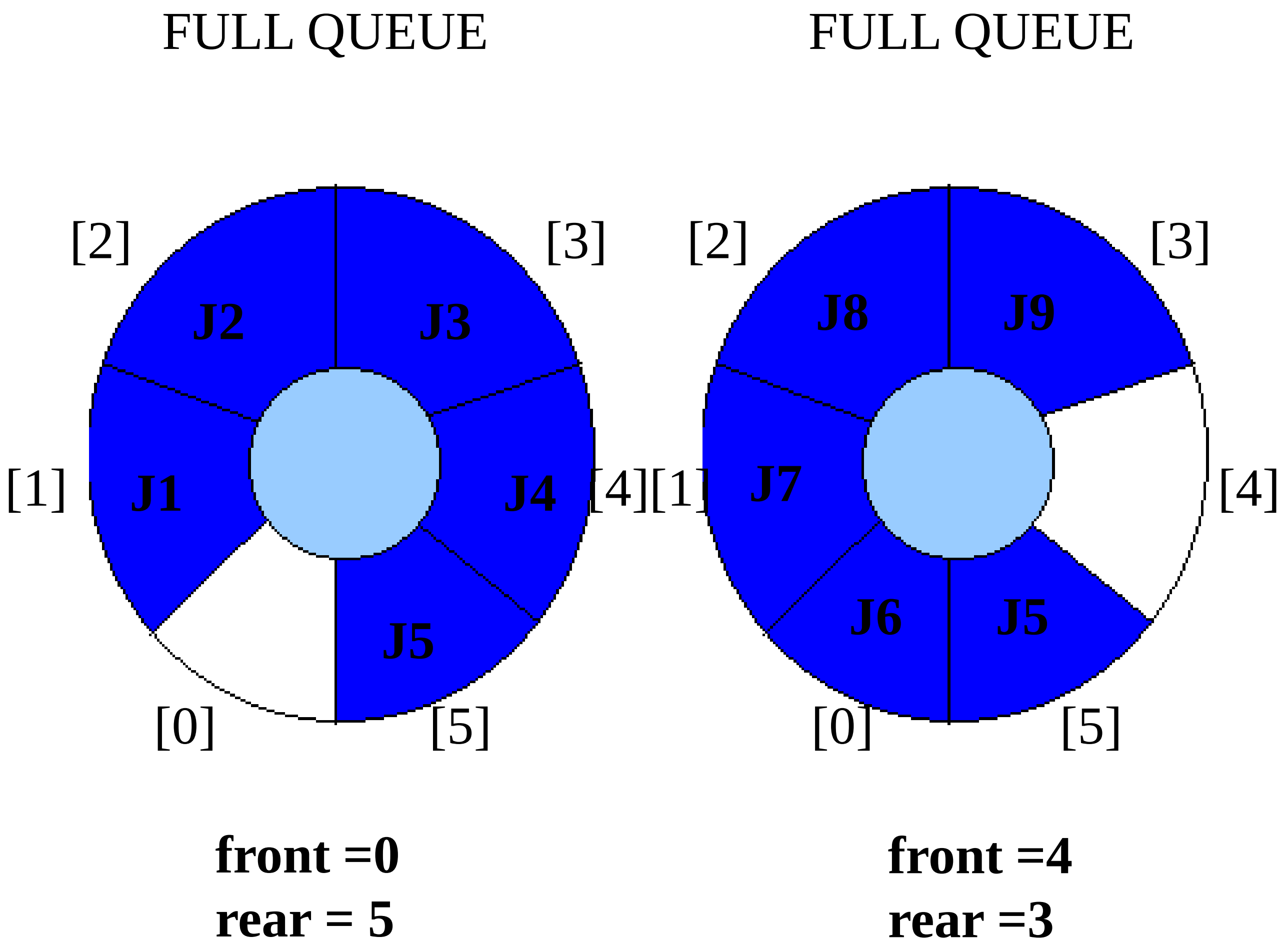


Figure 3.7: Full circular queues and then we remove the item

Add to a circular queue

```
void addq(element item)
{
/* add an item to the queue */
    rear = (rear +1) % MAX_QUEUE_SIZE;
     if (front == rear) /* reset rear and print error */
      queueFull();
   }
     queue[rear] = item;
}
```

Program 3.7: Add to a circular queue

Delete from a circular queue

```
element deleteq()
{
   element item;
   /* remove front element from the queue and put it in item */
       if (front == rear)
          return queueEmpty( );
                    /* queue_empty returns an error key */
      front = (front+1) % MAX_QUEUE_SIZE;
      return queue[front];
}
```

Program 3.8: Delete from a circular queue

A Mazing Problem

entrance →

0	1	0	0	0	1	1	0	0	0	1	1	1	1	1
1	0	0	0	1	1	0	1	1	1	0	0	1	1	1
0	1	1	0	0	0	0	1	1	1	1	0	0	1	1
1	1	0	1	1	1	1	0	1	1	0	1	1	0	0
1	1	0	1	0	0	1	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	0	1	0	0	1	0	1
0	1	1	1	1	0	0	1	1	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1	1	1	1	0	1
1	1	0	0	0	1	1	0	1	1	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	1	1	1	1	0
0	1	0	0	1	1	1	1	1	0	1	1	1	1	0

→ exit

1: blocked path

0: through path

***Figure 3.8:** An example maze

a possible representation

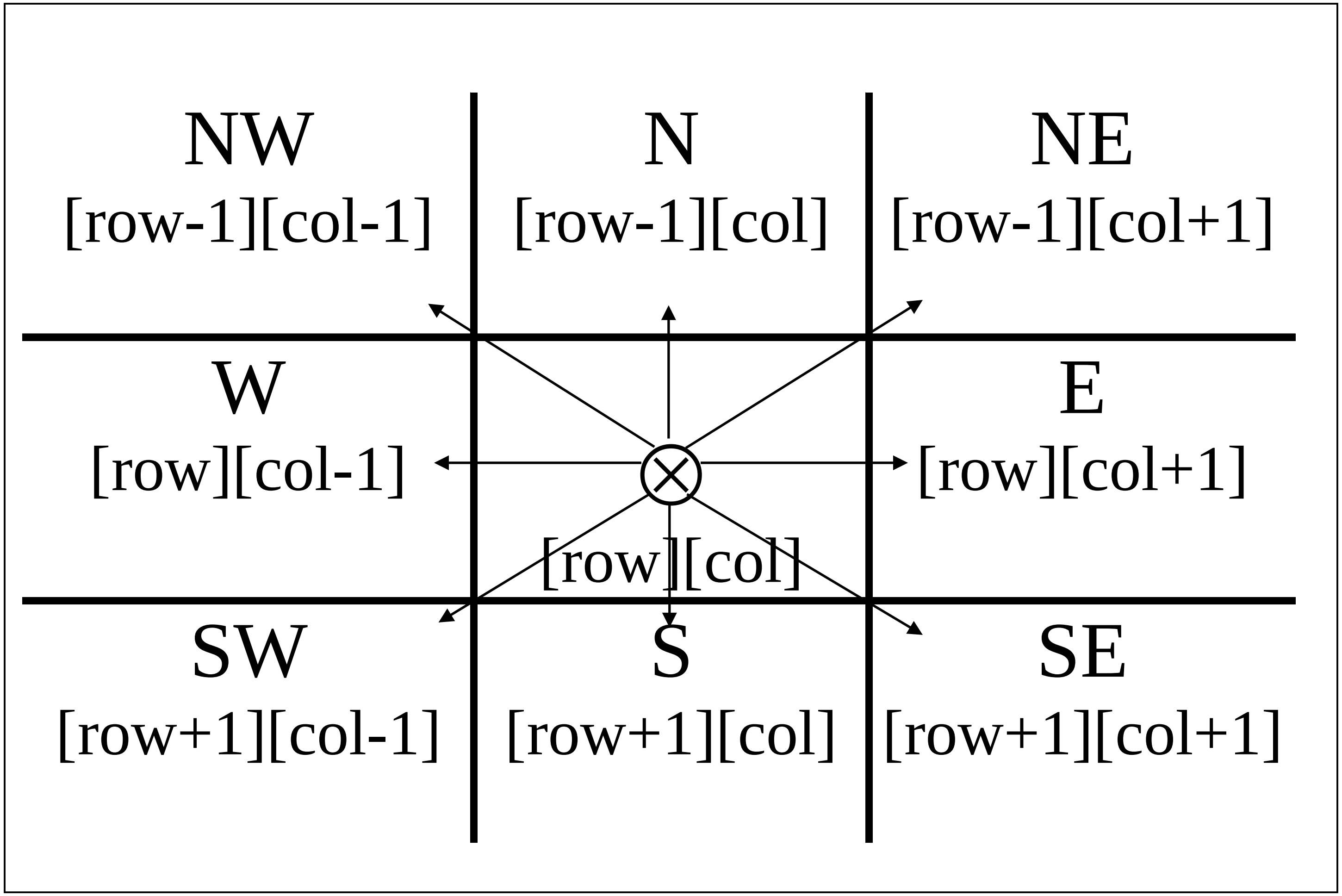


Figure 3.9: Allowable moves

a possible implementation

```
typedef struct {
        short int vert;
        short int horiz;
        } offsets;
offsets move[8]; /*array of moves for each direction*/
```

```
next_row = row + move[dir].vert;
next_col  = col  +  move[dir].horiz;
```

Name	Dir	move[dir].vert	move[dir].horiz
N	0	-1	0
NE	1	-1	1
E	2	0	1
SE	3	1	1
S	4	1	0
SW	5	1	-1
W	6	0	-1
NW	7	-1	-1

Use stack to keep pass history

```
#define MAX_STACK_SIZE 100
        /*maximum stack size*/
typedef struct {
        short int row;
        short int col;
        short int dir;
        } element;
element stack[MAX_STACK_SIZE];
```

A Mazing Problem

- 設計一個地圖，用stack記錄從起點走到終點的路徑
- 實行步驟:
  - initMap()
    - 初始地圖map[][]
    - 初始標記mark[][]
    - 初始方位(八方位)
  - path()

起點

1	1	1	1	1	1
1	0	0	1	0	1
1	0	0	0	1	1
1	0	1	1	1	1
1	0	0	0	0	1
1	1	1	1	1	1

map[6][6] **終點**

A Mazing Problem

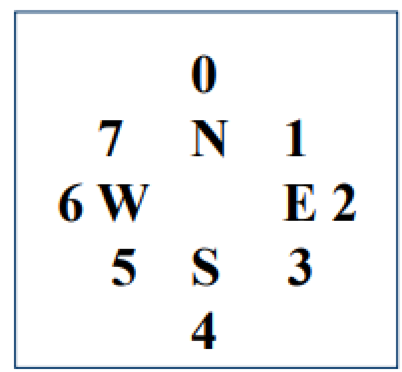
- init
- 0是路(可以走的)，1是牆(不能走的)

1	1	1	1	1	1
1	0	0	1	0	1
1	0	0	0	1	1
1	0	1	1	1	1
1	0	0	0	0	1
1	1	1	1	1	1

map[6][6]

1	1	1	1	1	1
1	0	0	0	0	1
1	0	0	0	0	1
1	0	0	0	0	1
1	0	0	0	0	1
1	1	1	1	1	1

mark[6][6]



八方位

A Mazing Problem

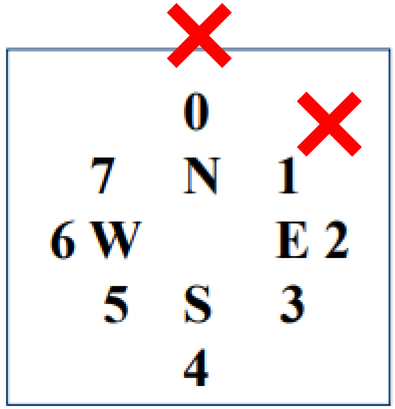
- 第一步
- 從起點(1, 1)開始，mark標記已走過，直到方向2有路且沒有走過

1	1	1	1	1	1
1	*	0	1	0	1
1	0	0	0	1	1
1	0	1	1	1	1
1	0	0	0	0	1
1	1	1	1	1	1

map[6][6]

1	1	1	1	1	1
1	*	0	0	0	1
1	0	0	0	0	1
1	0	0	0	0	1
1	0	0	0	0	1
1	1	1	1	1	1

mark[6][6]



八方位

A Mazing Problem

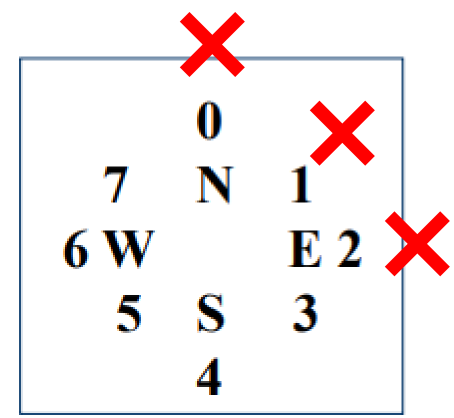
- 第二步
- 走向(1, 2)，mark標記已走過，直到方向3有路且沒有走過

1	1	1	1	1	1
1	*	*	1	0	1
1	0	0	0	1	1
1	0	1	1	1	1
1	0	0	0	0	1
1	1	1	1	1	1

map[6][6]

1	1	1	1	1	1
1	*	*	0	0	1
1	0	0	0	0	1
1	0	0	0	0	1
1	0	0	0	0	1
1	1	1	1	1	1

mark[6][6]



八方位

A Mazing Problem

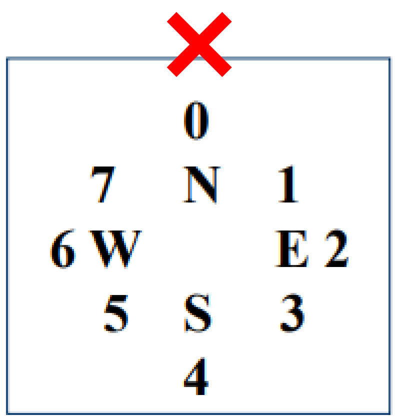
- 第三步
- 走向(2, 3)，mark標記已走過，直到方向1有路且沒有走過

1	1	1	1	1	1
1	*	*	1	0	1
1	0	0	*	1	1
1	0	1	1	1	1
1	0	0	0	0	1
1	1	1	1	1	1

map[6][6]

1	1	1	1	1	1
1	*	*	0	0	1
1	0	0	*	0	1
1	0	0	0	0	1
1	0	0	0	0	1
1	1	1	1	1	1

mark[6][6]



八方位

A Mazing Problem

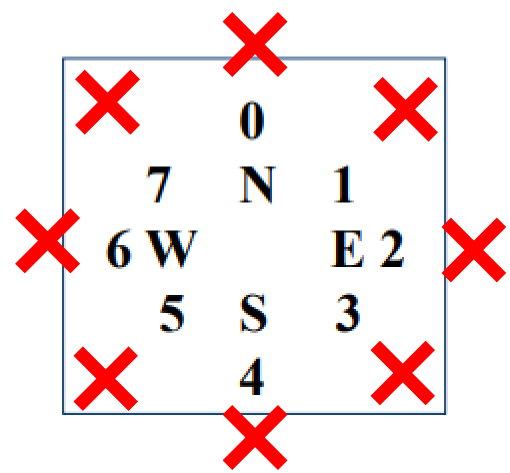
- 第四步
- 走向(1, 4)，mark標記已走過，所有方向都沒路，退回上一步

1	1	1	1	1	1
1	*	*	1	0	1
1	0	0	*	1	1
1	0	1	1	1	1
1	0	0	0	0	1
1	1	1	1	1	1

map[6][6]

1	1	1	1	1	1
1	*	*	0	*	1
1	0	0	*	0	1
1	0	0	0	0	1
1	0	0	0	0	1
1	1	1	1	1	1

mark[6][6]



八方位

A Mazing Problem

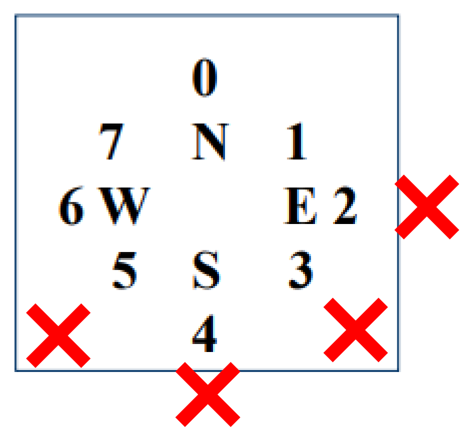
- 第五步
- 走向(2, 3)，從方向2開始繼續找路，直到方向6有路且沒有走過

1	1	1	1	1	1
1	*	*	1	0	1
1	0	0	*	1	1
1	0	1	1	1	1
1	0	0	0	0	1
1	1	1	1	1	1

map[6][6]

1	1	1	1	1	1
1	*	*	0	*	1
1	0	0	*	0	1
1	0	0	0	0	1
1	0	0	0	0	1
1	1	1	1	1	1

mark[6][6]



八方位

A Mazing Problem

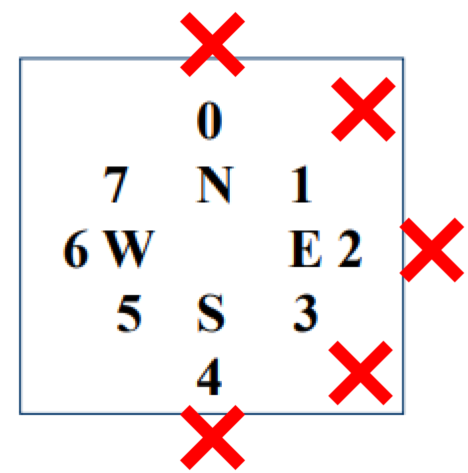
- 第六步
- 走向(2, 2)，mark標記已走過，直到方向5有路且沒有走過

1	1	1	1	1	1
1	*	*	1	0	1
1	0	*	*	1	1
1	0	1	1	1	1
1	0	0	0	0	1
1	1	1	1	1	1

map[6][6]

1	1	1	1	1	1
1	*	*	0	*	1
1	0	*	*	0	1
1	0	0	0	0	1
1	0	0	0	0	1
1	1	1	1	1	1

mark[6][6]



八方位

A Mazing Problem

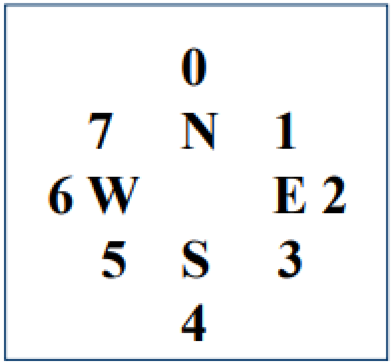
- 以此類推

1	1	1	1	1	1
1	*	*	1	0	1
1	0	*	*	1	1
1	*	1	1	1	1
1	0	0	0	0	1
1	1	1	1	1	1

map[6][6]

1	1	1	1	1	1
1	*	*	0	*	1
1	0	*	*	0	1
1	*	0	0	0	1
1	0	0	0	0	1
1	1	1	1	1	1

mark[6][6]



八方位

A Mazing Problem

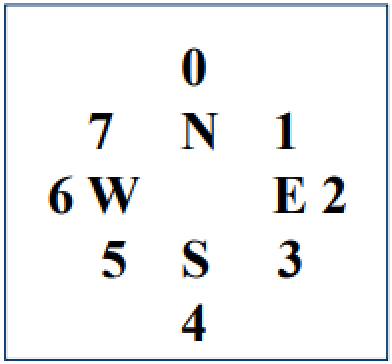
- 以此類推

1	1	1	1	1	1
1	*	*	1	0	1
1	0	*	*	1	1
1	*	1	1	1	1
1	0	0	0	0	1
1	1	1	1	1	1

map[6][6]

1	1	1	1	1	1
1	*	*	0	*	1
1	*	*	*	0	1
1	*	0	0	0	1
1	0	0	0	0	1
1	1	1	1	1	1

mark[6][6]



八方位

A Mazing Problem

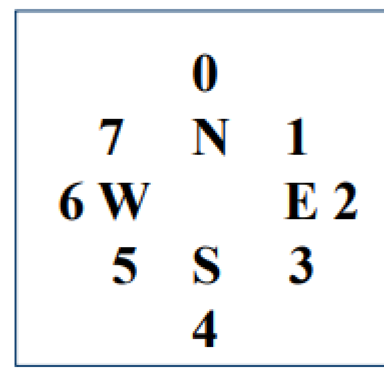
- 以此類推

1	1	1	1	1	1
1	*	*	1	0	1
1	0	*	*	1	1
1	*	1	1	1	1
1	0	0	0	0	1
1	1	1	1	1	1

map[6][6]

1	1	1	1	1	1
1	*	*	0	*	1
1	*	*	*	0	1
1	*	0	0	0	1
1	0	0	0	0	1
1	1	1	1	1	1

mark[6][6]



八方位

A Mazing Problem

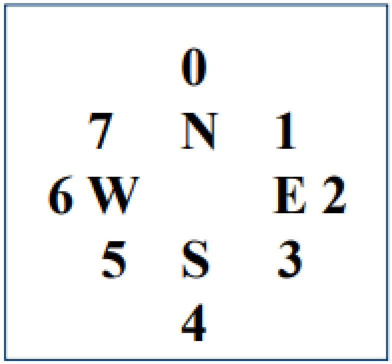
- 以此類推

1	1	1	1	1	1
1	*	*	1	0	1
1	0	*	*	1	1
1	*	1	1	1	1
1	0	*	0	0	1
1	1	1	1	1	1

map[6][6]

1	1	1	1	1	1
1	*	*	0	*	1
1	*	*	*	0	1
1	*	0	0	0	1
1	0	*	0	0	1
1	1	1	1	1	1

mark[6][6]



八方位

A Mazing Problem

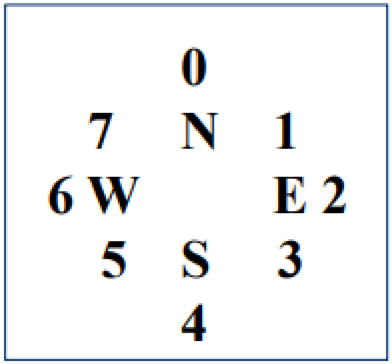
- 以此類推

1	1	1	1	1	1
1	*	*	1	0	1
1	0	*	*	1	1
1	*	1	1	1	1
1	0	*	*	0	1
1	1	1	1	1	1

map[6][6]

1	1	1	1	1	1
1	*	*	0	*	1
1	*	*	*	0	1
1	*	0	0	0	1
1	0	*	*	0	1
1	1	1	1	1	1

mark[6][6]



八方位

A Mazing Problem

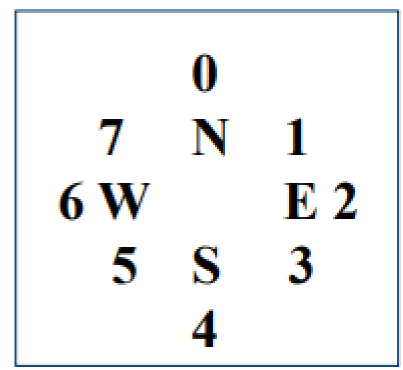
- 以此類推，最後走到終點

1	1	1	1	1	1
1	*	*	1	0	1
1	0	*	*	1	1
1	*	1	1	1	1
1	0	*	*	*	1
1	1	1	1	1	1

map[6][6]

1	1	1	1	1	1
1	*	*	0	*	1
1	*	*	*	0	1
1	*	0	0	0	1
1	0	*	*	*	1
1	1	1	1	1	1

mark[6][6]



八方位

```
Initialize a stack to the maze's entrance coordinates and direction
to north;
while (stack is not empty){
   /* move to position at top of stack */
<row, col, dir> = delete from top of stack;
while (there are more moves from current position) {
     <next_row, next_col > = coordinates of next move;
     dir = direction of move;
     if ((next_row == EXIT_ROW)&& (next_col == EXIT_COL))
         success; /* find out the destination */
     if (maze[next_row][next_col] == 0 &&
          mark[next_row][next_col] == 0) {
```

```
    /* legal move and haven't been there */
       mark[next_row][next_col] = 1;
       /* save current position and direction */
       add <row, col, dir> to the top of the stack;
       row = next_row;
       col  = next_col;
       dir  = north;
    }
  }
}
printf("No path found\n");
```

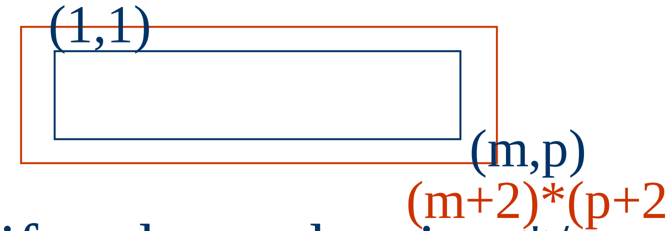
***Program 3.11:** Initial maze algorithm

The size of a stack?

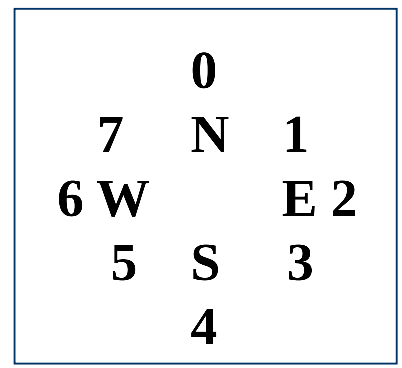
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{m*p}$$

The worst case complexity of the algorithm is $O(mp)$

Figure 3.11: Simple maze with a long path




```
void path (void)
{
/* output a path through the  maze if such a path exists */
    int i, row, col, next_row, next_col, dir, found = FALSE;
    element position;
    mark[1][1] = 1; top =0;
    stack[0].row = 1; stack[0].col = 1; stack[0].dir = 1;
    while (top > -1 && !found) {
       position = pop();
       row = position.row; col = position.col;
       dir  = position.dir;
       while (dir < 8 && !found) {
              /*move in direction dir  */
              next_row = row + move[dir].vert;
              next_col  = col + move[dir].horiz;
```



```
    if (next_row==EXIT_ROW && next_col==EXIT_COL)
        found = TRUE;    //Find the Exit
    else if ( !maze[next_row][next_col] &&
              !mark[next_row][next_col] {
        mark[next_row][next_col] = 1;
        position.row = row;
        position.col = col;
        position.dir = ++dir;
        push(position);
        row = next_row;
        col = next_col;
        dir = 0;
     }
     else ++dir;    // Change to different directions
  }
}
```

```
    if (found) {
        printf("The path is :\n");
        printf("row col\n");
        for (i = 0; i <= top; i++)
              printf(" %2d%5d", stack[i].row, stack[i].col);
        printf("%2d%5d\n", row, col);
        printf("%2d%5d\n", EXIT_ROW, EXIT_COL);
     }
   else printf("The maze does not have a path\n");
}
```

Program 3.12: Maze search function

Evaluation of Expressions

$X = a / b - c + d * e - a * c$

$a = 4, b = c = 2, d = e = 3$

How to generate the machine instructions corresponding to a given expression?

Interpretation 1:

$((4/2)-2)+(3*3)-(4*2)=0 + 9 - 8=1$

Interpretation 2:

$(4/(2-2+3))*(3-4)*2=(4/3)*(-1)*2=-2.66666\cdots$

precedence rule + associative rule

Token	Operator	Precedence[1]	Associativity
( ) [ ] -> .	function call array element struct or union member	17	left-to-right
-- ++	increment, decrement[2]	16	left-to-right
-- ++ ! - - + & * sizeof	decrement, increment[3] logical not one's complement unary minus or plus address or indirection size (in bytes)	15	right-to-left
(type)	type cast	14	right-to-left
* / %	mutiplicative	13	Left-to-right

+ -	binary add or subtract	12	left-to-right
<< >>	shift	11	left-to-right
> >= < <=	relational	10	left-to-right
== !=	equality	9	left-to-right
&	bitwise and	8	left-to-right
^	bitwise exclusive or	7	left-to-right
|	bitwise or	6	left-to-right
&&	logical and	5	left-to-right
XX	logical or	4	left-to-right

?:	conditional	3	right-to-left
= += -= /= *= %= <<= >>= &= ^= x=	assignment	2	right-to-left
,	comma	1	left-to-right

1.The precedence column is taken from Harbison and Steele.
2.Postfix form
3.prefix form

Figure 3.12: Precedence hierarchy for C

user **compiler**

Infix	Postfix
2+3*4	234*+
a*b+5	ab*5+
(1+2)*7	12+7*
a*b/c	ab*c/
(a/(b-c+d))*(e-a)*c	abc-d+/ea-*c*
a/b-c+d*e-a*c	ab/c-de*+ac*-

Figure 3.13: Infix and postfix notation

Postfix: no parentheses, no precedence

Token	Stack [0]	[1]	[2]	Top
6	6			0
2	6	2		1
/	6/2			0
3	6/2	3		1
-	6/2-3			0
4	6/2-3	4		1
2	6/2-3	4	2	2
*	6/2-3	4*2		1
+	6/2-3+4*2			0

Figure 3.14: Postfix evaluation

Infix to postfix

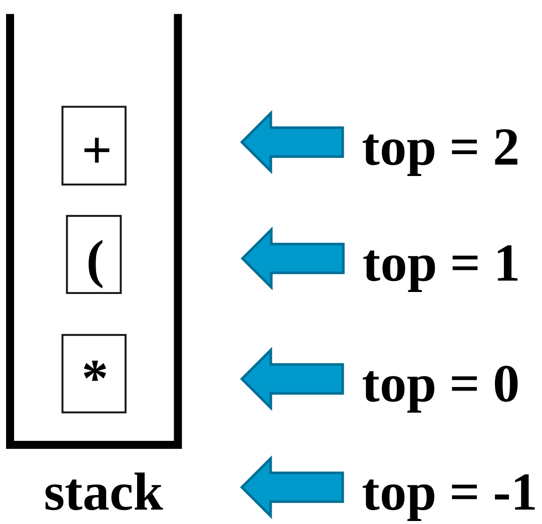
- 輸入infix，輸出postfix
- 主要二個function
  1. token to string: getToken()
  2. infix to postfix: postfix()

Infix to postfix

- input: a*(b+c)*d
- token
- output

a * (b + c) * d

a b c + * d *



Goal: infix --> postfix

Assumptions:
operators: +, -, *, /, %
operands: single digit integer

```
#define MAX_STACK_SIZE 100 /* maximum stack size */
#define MAX_EXPR_SIZE 100 /* max size of expression */
typedef enum{1paran, rparen, plus, minus, times, divide,
                    mod, eos, operand} precedence;
int stack[MAX_STACK_SIZE]; /* global stack */
char expr[MAX_EXPR_SIZE]; /* input string */
```


eos : end of string

```
int eval(void)
{
/* evaluate a postfix expression, expr, maintained as a
   global variable, '\0' is the the end of the expression.
   The stack and top of the stack are global variables.
   get_token is used to return the token type and
   the character symbol. Operands are assumed to be single
   character digits */
  precedence token;
  char symbol;
  int op1, op2;
  int n = 0;  /* counter for the expression string */
  int top = -1;
  token = get_token(&symbol, &n);
  while (token != eos)
   {
     if (token == operand)
         push(symbol-'0');   /* stack insert */
                              /*  ASCII 0*/
```

exp: character array

```
  else {
        /* remove two operands, perform operation, and
           return result to the stack */
     op2 = pop();  /* stack delete */
     op1 = pop();
     switch(token) {
         case plus: push(op1+op2); break;
         case minus: push(op1-op2); break;
         case times: push(op1*op2); break;
         case divide: push(op1/op2); break;
         case mod: push(op1%op2);
         }
       }
  token = get_token (&symbol, &n);
 }
return pop(); /* return result */
}
```

Program 3.13: Function to evaluate a postfix expression

Infix to postfix

```
precedence get_token(char *symbol, int *n) {
    *symbol = expr[(*n)++];
    switch(*symbol) {
        case '(': return lparen;
        case ')': return rparen;
        case '+': return plus;
        case '-': return minus;
        case '/': return divide;
        case '*': return times;
        case '%': return mod;
        case ' ': return blank; // 空白
        case '\0': return eos; // 空字符
        default: return operand;
    }
}
```

Infix to Postfix Conversion
(Intuitive Algorithm)

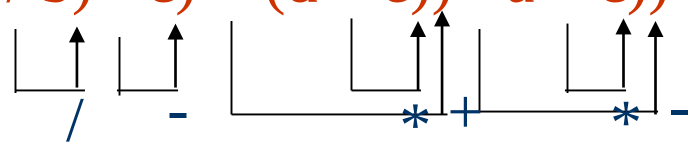
(1) Fully parenthesize expression

a / b - c + d * e - a * c -->

((((a / b) - c) + (d * e)) - a * c))

(2) All operators replace their corresponding right parentheses.

((((a / b) - c) + (d * e)) - a * c))



(3) Delete all parentheses.

ab/c-de*+ac*-

two passes

The orders of operands in infix and postfix are the same.

a + b * c

Token	Stack [0]	Stack [1]	Stack [2]	Top	Output
a				-1	a
+	+			0	a
b	+			0	ab
*	+	*		1	ab
c	+	*		1	abc
eos				-1	abc*+

Figure 3.15: Translation of a+b*c to postfix

$a *_1 (b + c) *_2 d$

Token	Stack [0]	Stack [1]	Stack [2]	Top	Output
a				-1	a
$*_1$	$*_1$			0	a
(	$*_1$	(		1	a
b	$*_1$	(		1	ab
+	$*_1$	(	+	2	ab
c	$*_1$	(	+	2	abc
)	$*_1$	match)		0	abc+
$*_2$	$*_2$	$*_1 = *_2$		0	$abc+*_1$
d	$*_2$			0	$abc+*_1d$
eos	$*_2$			0	$abc+*_1d*_2$

Figure 3.16: Translation of a*(b+c)*d to postfix

Rules

(1) Operators are **taken out** of the stack as long as their in-stack precedence (isp) is **higher than or equal to** the incoming precedence (icp) of the new operator.

(2) **“(_“** has **low** in-stack precedence, and **high** incoming precedence.

	(	)	+	-	*	/	%	eos
isp	0	19	12	12	13	13	13	0
icp	20	19	12	12	13	13	13	0

```
precedence stack[MAX_STACK_SIZE];
/* isp and icp arrays -- index is value of precedence
lparen, rparen, plus, minus, times, divide, mod, eos */
static int isp [ ] = {0, 19, 12, 12, 13, 13, 13, 0};
static int icp [ ] = {20, 19, 12, 12, 13, 13, 13, 0};
```

isp: in-stack precedence
icp: incoming precedence

```
void postfix(void)
{
/* output the postfix of the expression. The expression
    string, the stack, and top are global */
   char symbol;
   precedence token;
   int n = 0;
   int top = 0; /* place eos on stack */
   stack[0] = eos;
   for (token = get _token(&symbol, &n); token != eos;
                    token = get_token(&symbol, &n)) {
     if (token == operand)
        printf ("%c", symbol);
     else if (token == rparen ){
```

```
/*unstack tokens until left parenthesis */
while (stack[top] != lparen)
    print_token(pop());
pop(); /*discard the left parenthesis */
}
else{
/* remove and print symbols whose isp is greater
    than or equal to the current token's icp */
while(isp[stack[top]] >= icp[token] )
    print_token(pop(&top));
push(token);
}
}
while ((token = pop(&top)) != eos)
    print_token(token);
print("\n");
}
```

$\theta(n)$

f(n)=θ(g(n)) iff there exist positive constants c_1, c_2, and n_0 such that $c_1 g(n) \le f(n) \le c_2 g(n)$ for all n, $n \ge n_0$.

f(n)=θ(g(n)) iff g(n) is both an upper and lower bound on f(n).

***Program 3.15**: Function to convert from infix to postfix

Infix to postfix

```
void postfix(void) {
    char symbol;
    precedence token;
    int n = 0;
    top = 0;
    stack[0] = eos;
    for(token = get_token(&symbol, &n); token != eos;token = get_token(&symbol, &n)) {
        if(token != blank) {                    // token不是空白
            if(token == operand) {              // token是運算元
                *str++=symbol;
            }
            else if(token == rparen) {          // token是右括號
                *str++=' ';
                while(stack[top] != lparen){ // 把stack輸出直到遇到左括號
                    print_token(pop());
                }
                pop();                          // 輸出左括號
            }
            else {                              // token是plus, minus, times, divide, mod
                *str++=' ';
                while(isp[stack[top]] >= icp[token])
                    print_token(pop());
                push(token);
            }
        }
    }
    *str++=' ';
    while((token = pop()) != eos)   // 把stack清空
        print_token(token);
    *str++='\0';
    printf("\n");
}
```

Infix	Prefix
a*b/c a/b-c+d*e-a*c a*(b+c)/d-g	/*abc -+-/abc*de*ac -/*a+bcdg

(1) evaluation
(2) transformation

***Figure 3.17:** Infix and postfix expressions

Multiple Stacks and Queues

Two stacks

m[0], m[1], …, m[n-2], m[n-1]

bottommost stack 1 → ← bottommost stack 2

More than two stacks (n)

memory is divided into *n* equal segments

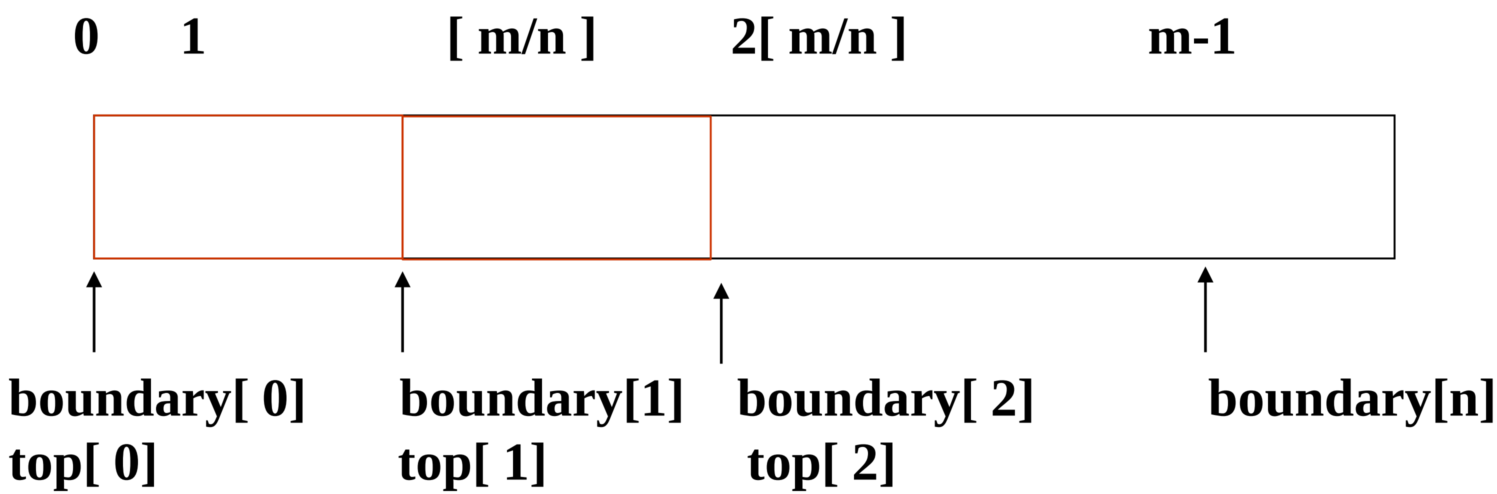
boundary[stack_no]

$0 \leq$ stack_no < MAX_STACKS

top[stack_no]

$0 \leq$ stack_no < MAX_STACKS

Initially, boundary[i]=top[i].



All stacks are empty and divided into roughly equal segments.

***Figure 3.18:** Initial configuration for *n* stacks in memory [m].

```
#define MEMORY_SIZE 100    /* size of memory */
#define MAX_STACK_SIZE 100
            /* max number of stacks plus 1 */
/* global memory declaration */
element memory[MEMORY_SIZE];
int top[MAX_STACKS];
int boundary[MAX_STACKS];
int n;  /* number of stacks entered by the user */
```

```
top[0] = boundary[0] = -1;
for (i = 1; i < n; i++)
  top[i] =boundary[i] =(MEMORY_SIZE/n)*i;
boundary[n] = MEMORY_SIZE-1;
```

```
void push(int i, element item)
{
   /* add an item to the ith stack */
   if (top[i] == boundary [i+1])
      stackFull(i);
    memory[++top[i]] = item;
}
```

but it may have unused storage

*__Program 3.16:__Add an item to the stack *stack-no*

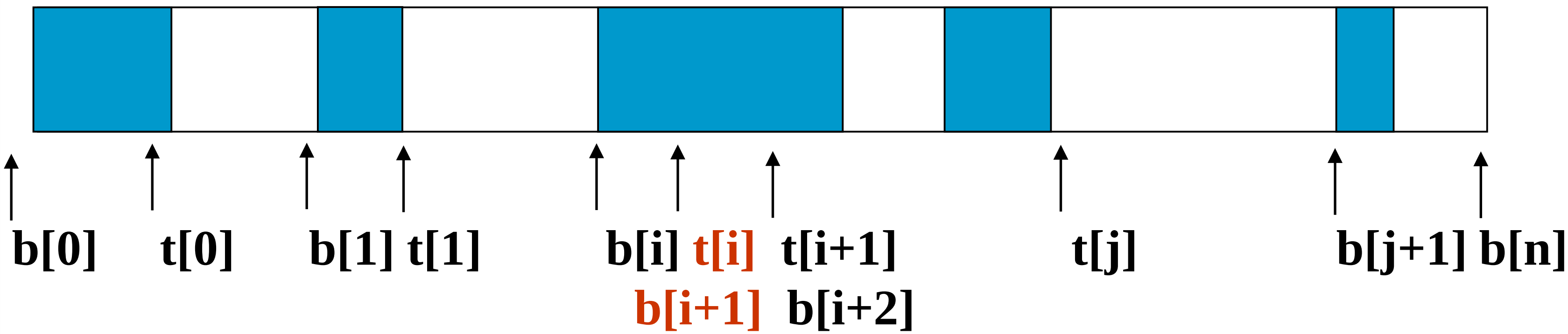
```
element pop(int i)
{
   /* remove top element from the ith stack */
   if (top[i] == boundary[i])
      return stackEmpty(i);
   return memory[top[i]--];
}
```

***Program 3.17:**Delete an *item* from the stack *stack-no*

Find j, stack_no < j < n (往右)

such that top[j] < boundary[j+1]

or, 0 ≤ j < stack_no (往左)



meet

b=boundary, t=top

← 往左或右找一個空間 →

***Figure 3.19:** Configuration when stack i meets stack i+1, but the memory is not full (p.130)