CHAPTER 3

STACKS AND QUEUES

All the programs in this file are selected from

Ellis Horowitz, Sartaj Sahni, and Susan Anderson-Freed
"Fundamentals of Data Structures in C",

Stack: a Last-In-First-Out (LIFO) list

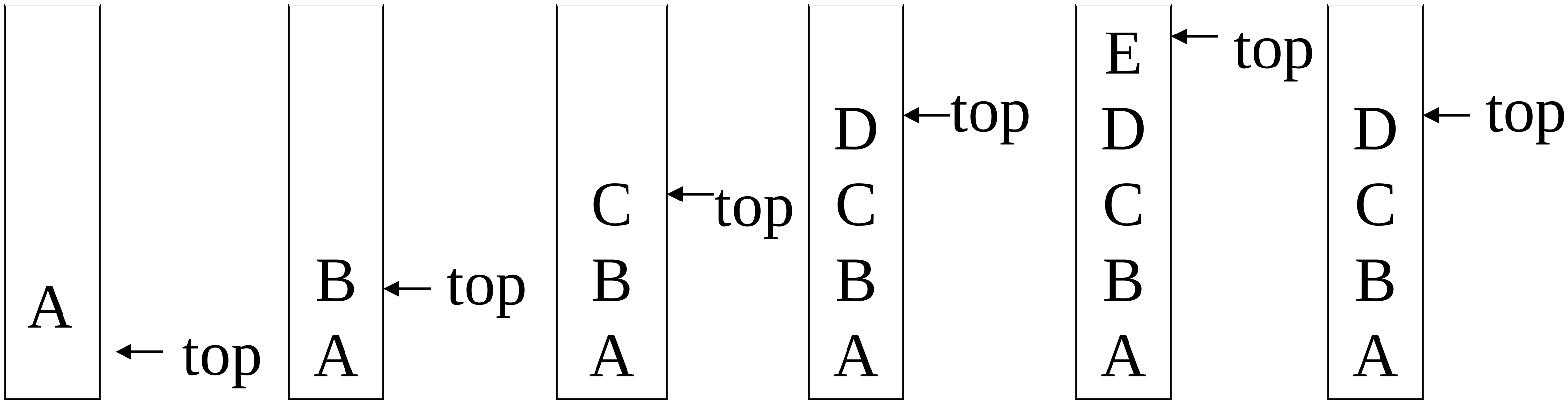
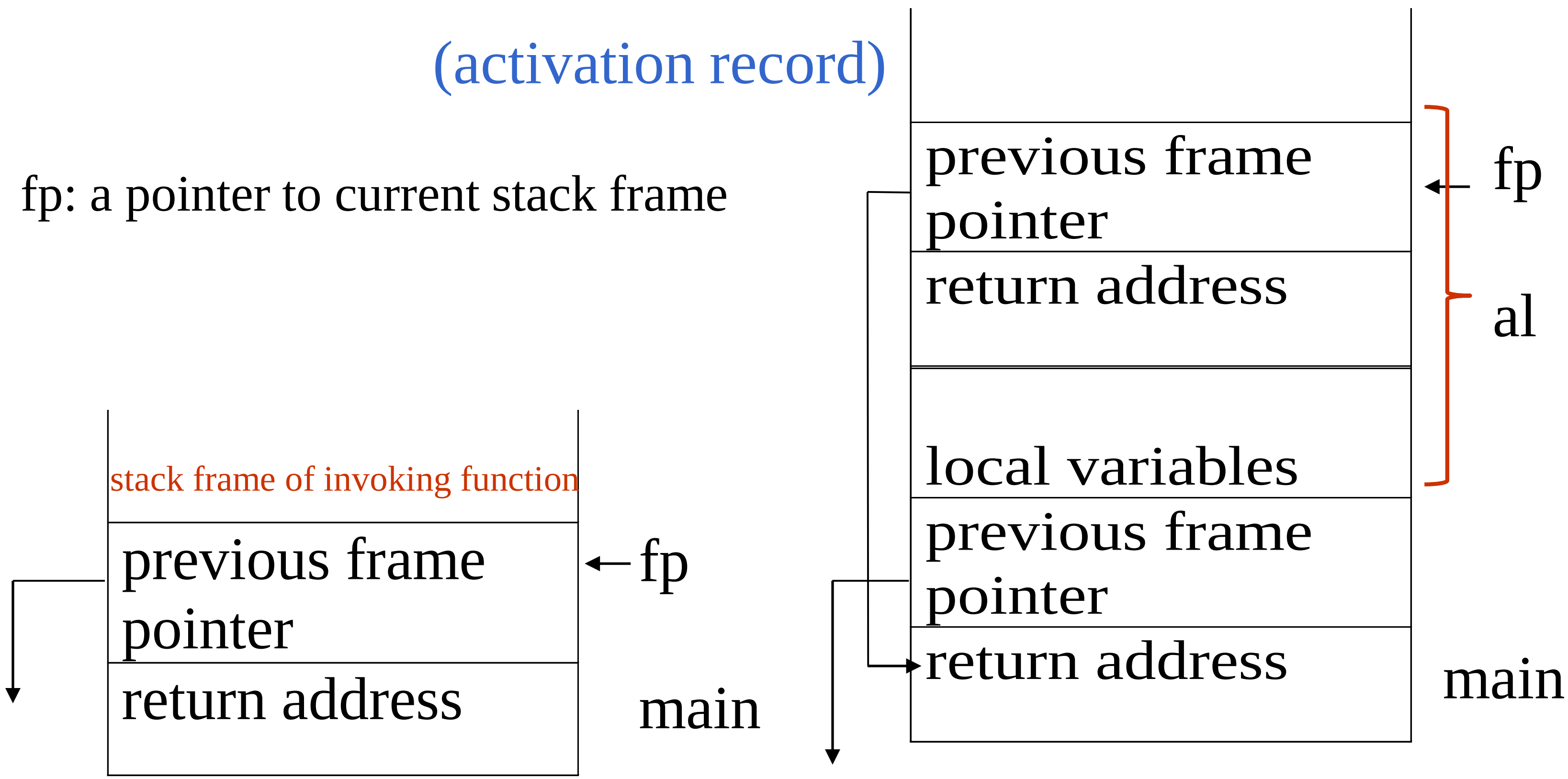


Figure 3.1: Inserting and deleting elements in a stack

An application of stack: stack frame of function call

(activation record)

fp: a pointer to current stack frame



system stack before a1 is invoked

(a)

system stack after a1 is invoked

(b)

Figure 3.2: System stack after function call a1

abstract data type for stack

structure *Stack* is

objects: a finite ordered list with zero or more elements.

functions:

for all *stack* ∈ *Stack*, *item* ∈ *element*, *max_stack_size* ∈ positive integer

Stack CreateS(*max_stack_size*) ::=
    create an empty stack whose maximum size is *max_stack_size*

Boolean IsFull(*stack*, *max_stack_size*) ::=
    if (number of elements in *stack* == *max_stack_size*)
    return TRUE
    else return FALSE

Stack Push(*stack*, *item*) ::=
    if (IsFull(*stack*)) *stack_full*
    else insert *item* into top of *stack* and **return**

Boolean IsEmpty(*stack*) ::=

if(*stack* == CreateS(*max_stack_size*))
return TRUE
else return FALSE

Element Pop(*stack*) ::=

if(IsEmpty(*stack*)) **return**
else remove and return the *item* on the top of the stack.

Structure 3.1: Abstract data type *Stack*

Implementation: using array

Stack **CreateS(max_stack_size)** ::=

```
#define MAX_STACK_SIZE 100 /* maximum stack size */
typedef struct {
        int key;
        /* other fields */
        } element;
element stack[MAX_STACK_SIZE];
int top = -1;
```

Boolean **IsEmpty(Stack)** ::= top< 0;

Boolean **IsFull(Stack)** ::= top >= MAX_STACK_SIZE-1;

Add to a stack

```
void push(int *top, element item)
{
 /* add an item to the global stack */
    if (*top >= MAX_STACK_SIZE-1)  {
         stack_full( );
         return;
    }
    stack[++*top] = item;
}
```

Program 3.1: Add to a stack

Delete from a stack

```
element pop(int *top)
{
 /* return the top element from the stack */
     if (*top == -1)
         return stack_empty( );  /* returns and error key */
     return stack[(*top)--];
 }
```

Program 3.2: Delete from a stack

Queue: a First-In-First-Out (FIFO) list

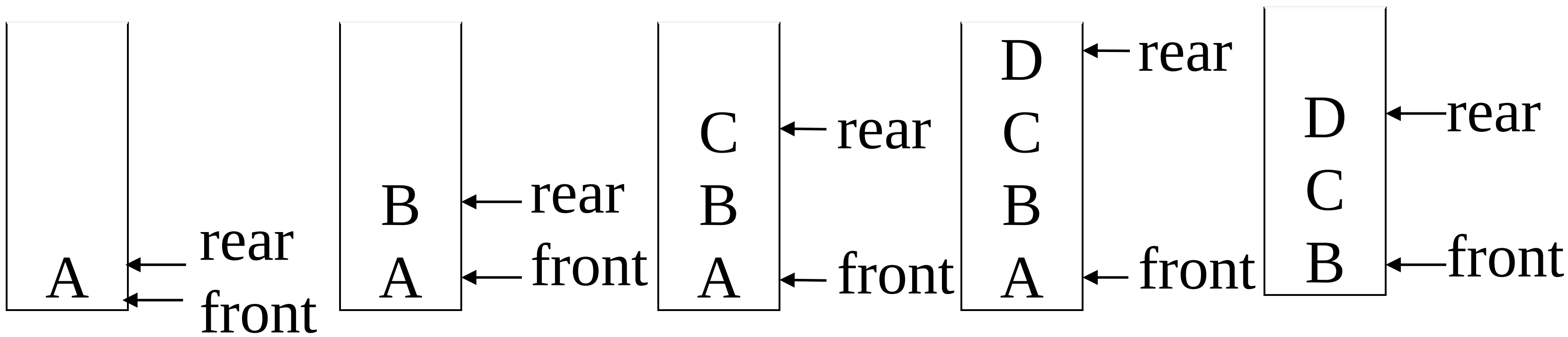


Figure 3.4: Inserting and deleting elements in a queue

Application: Job scheduling

front	rear	Q[0]	Q[1]	Q[2]	Q[3]	Comments
-1	-1					queue is empty
-1	0	J1				Job 1 is added
-1	1	J1	J2			Job 2 is added
-1	2	J1	J2	J3		Job 3 is added
0	2		J2	J3		Job 1 is deleted
1	2			J3		Job 2 is deleted

Figure 3.5: Insertion and deletion from a sequential queue

Abstract data type of queue

structure *Queue* is

objects: a finite ordered list with zero or more elements.

functions:

for all $queue \in Queue$, $item \in element$,

$max\_queue\_size \in$ positive integer

Queue CreateQ(*max_queue_size*) ::=

create an empty queue whose maximum size is *max_queue_size*

Boolean IsFullQ(*queue*, *max_queue_size*) ::=

if(number of elements in *queue* == *max_queue_size*)

return *TRUE*

else return *FALSE*

Queue AddQ(*queue*, *item*) ::=

if (IsFullQ(*queue*)) *queue_full*

else insert *item* at rear of *queue* and return *queue*

Boolean IsEmptyQ(*queue*) ::=
    if (*queue* ==CreateQ(*max_queue_size*))
    return *TRUE*
    else return *FALSE*
Element DeleteQ(*queue*) ::=
    if (IsEmptyQ(*queue*)) **return**
    else remove and return the *item* at front of queue.

Structure 3.2: Abstract data type *Queue*

Implementation 1: using array

```
Queue CreateQ(max_queue_size) ::=
# define MAX_QUEUE_SIZE 100/* Maximum queue size */
typedef struct {
                int key;
                /* other fields */
                } element;
element queue[MAX_QUEUE_SIZE];
int rear = -1;
int front = -1;
Boolean IsEmpty(queue) ::= front == rear
Boolean IsFullQ(queue) ::= rear == MAX_QUEUE_SIZE-1
```

Add to a queue

```
void addq(int rear, element item)
{
/* add an item to the queue */
    if (rear == MAX_QUEUE_SIZE-1) {
        queue_full( );
        return;
    }
    queue [++rear] = item;
}
```

Program 3.5: Add to a queue

Delete from a queue

```
element deleteq(int front, int rear)
{
/* remove element at the front of the queue */
    if ( front == rear)
        return queue_empty( );     /* return an error key */
    return queue [++ front];
}
```

Program 3.6: Delete from a queue

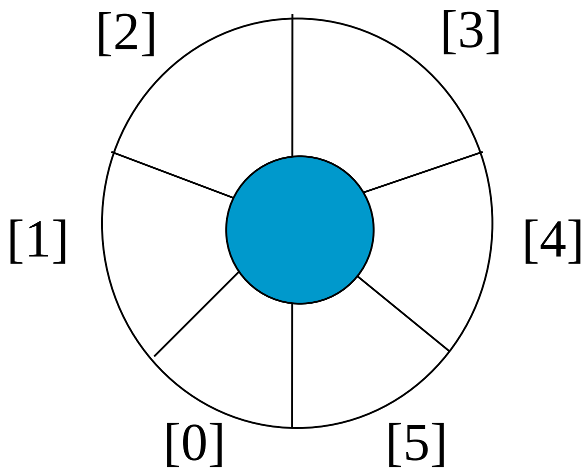
problem: there may be available space when IsFullQ is true i.e., movement is required.

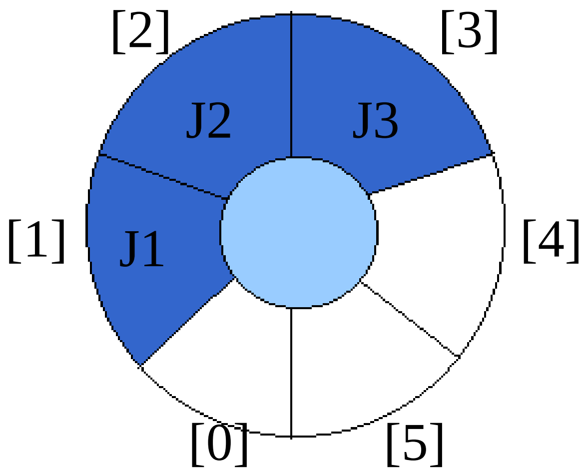
Implementation 2: regard an array as a circular queue

front: one position counterclockwise from the first element

rear: current end

EMPTY QUEUE





front = 0
rear = 0

front = 0
rear = 3

Figure 3.6: Empty and nonempty circular queues

Problem: one space is left when queue is full

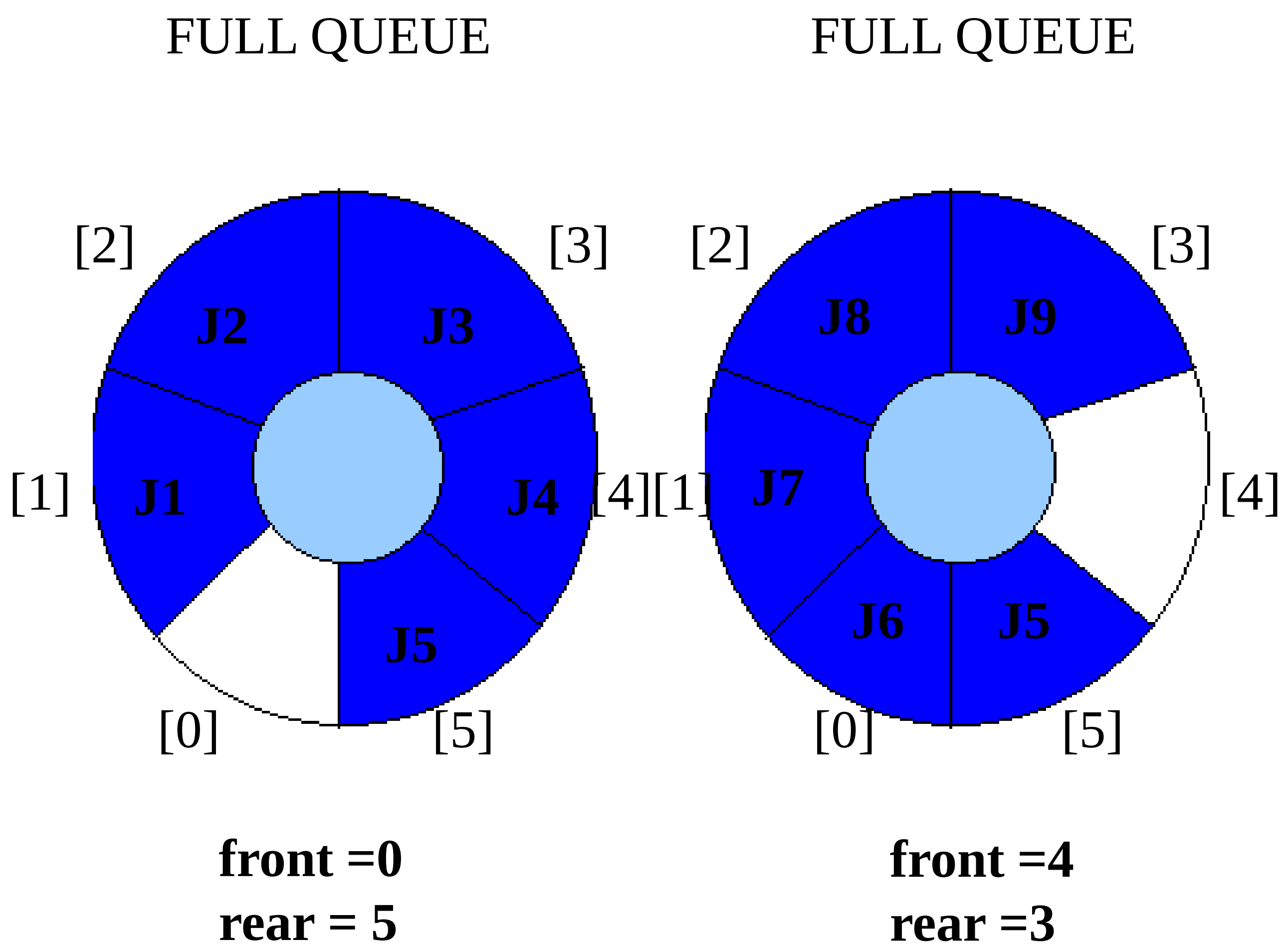


Figure 3.7: Full circular queues and then we remove the item

Add to a circular queue

```
void addq(element item)
{
/* add an item to the queue */
    rear = (rear +1) % MAX_QUEUE_SIZE;
     if (front == rear) /* reset rear and print error */
      queueFull();
   }
     queue[rear] = item;
}
```

Program 3.7: Add to a circular queue

Delete from a circular queue

```
element deleteq()
{
  element item;
  /* remove front element from the queue and put it in item */
     if (front == rear)
        return queueEmpty( );
                /* queue_empty returns an error key */
     front = (front+1) % MAX_QUEUE_SIZE;
     return queue[front];
}
```

Program 3.8: Delete from a circular queue

A Mazing Problem

entrance →

0	1	0	0	0	1	1	0	0	0	1	1	1	1	1
1	0	0	0	1	1	0	1	1	1	0	0	1	1	1
0	1	1	0	0	0	0	1	1	1	1	0	0	1	1
1	1	0	1	1	1	1	0	1	1	0	1	1	0	0
1	1	0	1	0	0	1	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	0	1	0	0	1	0	1
0	1	1	1	1	0	0	1	1	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1	1	1	1	0	1
1	1	0	0	0	1	1	0	1	1	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	1	1	1	1	0
0	1	0	0	1	1	1	1	1	0	1	1	1	1	0

→ exit

1: blocked path

0: through path

***Figure 3.8:** An example maze

a possible representation

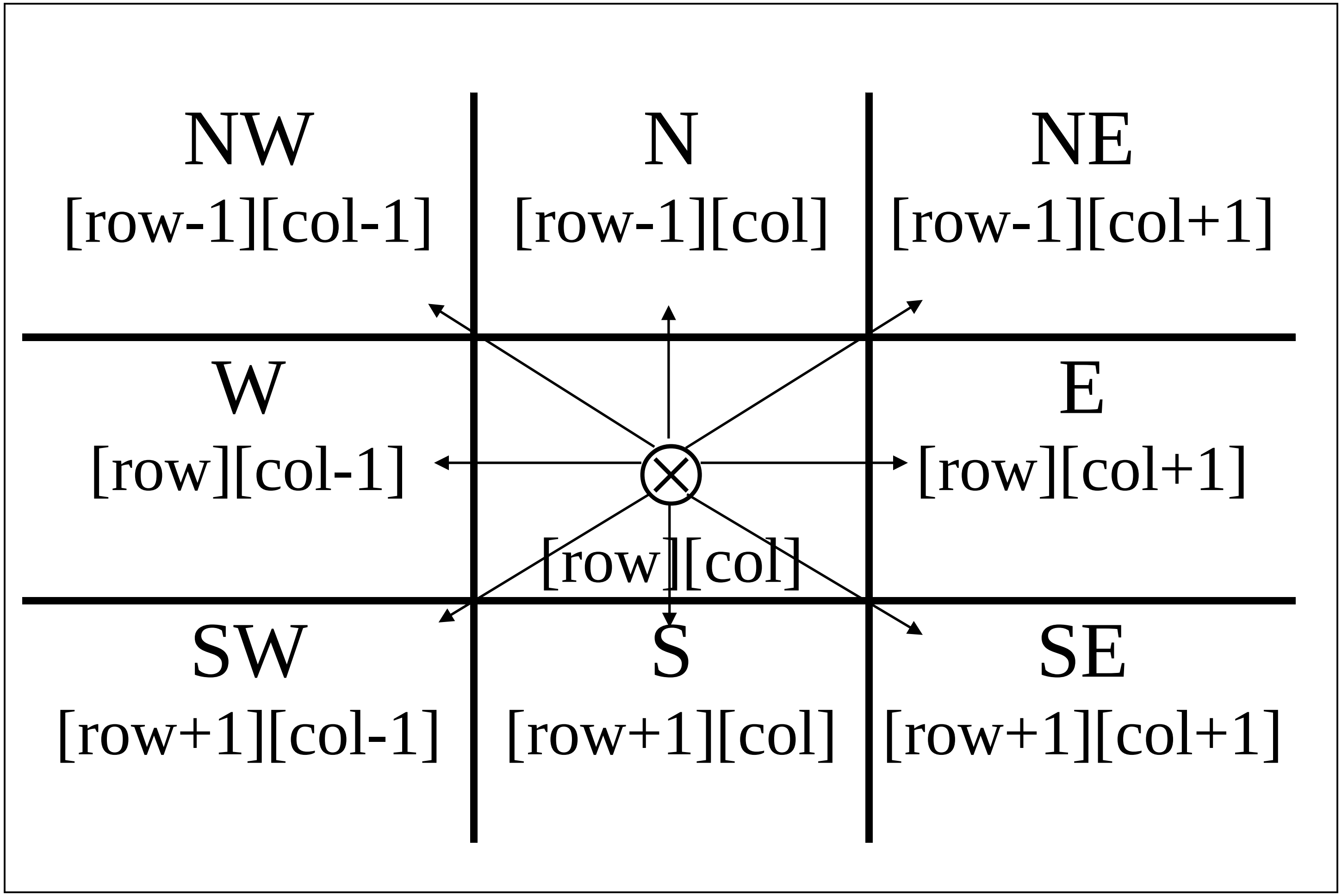


Figure 3.9: Allowable moves

a possible implementation

```
typedef struct {
        short int vert;
        short int horiz;
        } offsets;
offsets move[8]; /*array of moves for each direction*/
```

```
next_row = row + move[dir].vert;
next_col  = col  +  move[dir].horiz;
```

Name	Dir	move[dir].vert	move[dir].horiz
N	0	-1	0
NE	1	-1	1
E	2	0	1
SE	3	1	1
S	4	1	0
SW	5	1	-1
W	6	0	-1
NW	7	-1	-1

Use stack to keep pass history

```
#define MAX_STACK_SIZE 100
        /*maximum stack size*/
typedef struct {
        short int row;
        short int col;
        short int dir;
        } element;
element stack[MAX_STACK_SIZE];
```

```
Initialize a stack to the maze's entrance coordinates and direction
to north;
while (stack is not empty){
  /* move to position at top of stack */
<row, col, dir> = delete from top of stack;
while (there are more moves from current position) {
    <next_row, next_col > = coordinates of next move;
    dir = direction of move;
    if ((next_row == EXIT_ROW)&& (next_col == EXIT_COL))
        success; /* find out the destination */
    if (maze[next_row][next_col] == 0 &&
         mark[next_row][next_col] == 0) {
```

```
    /* legal move and haven't been there */
      mark[next_row][next_col] = 1;
      /* save current position and direction */
      add <row, col, dir> to the top of the stack;
      row = next_row;
      col  = next_col;
      dir  = north;
   }
  }
}
printf("No path found\n");
```

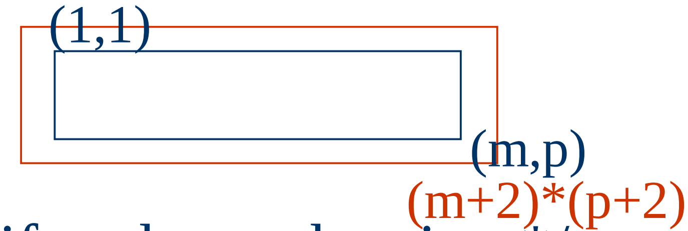
***Program 3.11:** Initial maze algorithm

The size of a stack?

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{m*p}$$

The worst case complexity of the algorithm is O(mp)

Figure 3.11: Simple maze with a long path




```
void path (void)
{
/* output a path through the  maze if such a path exists */
    int i, row, col, next_row, next_col, dir, found = FALSE;
    element position;
    mark[1][1] = 1; top =0;
    stack[0].row = 1; stack[0].col = 1; stack[0].dir = 1;
    while (top > -1 && !found) {
       position = pop();
       row = position.row; col = position.col;
       dir  = position.dir;
       while (dir < 8 && !found) {
              /*move in direction dir  */
              next_row = row + move[dir].vert;
              next_col  = col + move[dir].horiz;
```

0
7 N 1
6 W E 2
5 S 3
4

```
      if (next_row==EXIT_ROW && next_col==EXIT_COL)
          found = TRUE;    //Find the Exit
      else if ( !maze[next_row][next_col] &&
                   !mark[next_row][next_col] {
          mark[next_row][next_col] = 1;
          position.row = row;
          position.col = col;
          position.dir = ++dir;
          push(position);
          row = next_row;
          col = next_col;
          dir = 0;
       }
       else ++dir;    // Change to different directions
    }
}
```

```
    if (found) {
        printf("The path is :\n");
        printf("row col\n");
        for (i = 0; i <= top; i++)
            printf(" %2d%5d", stack[i].row, stack[i].col);
        printf("%2d%5d\n", row, col);
        printf("%2d%5d\n", EXIT_ROW, EXIT_COL);
     }
   else printf("The maze does not have a path\n");
}
```

Program 3.12: Maze search function

Evaluation of Expressions

X = a / b - c + d * e - a * c

a = 4, b = c = 2, d = e = 3

How to generate the machine instructions corresponding to a given expression?

Interpretation 1:

((4/2)-2)+(3*3)-(4*2)=0 + 9 - 8=1

Interpretation 2:

(4/(2-2+3))*(3-4)*2=(4/3)*(-1)*2=-2.66666…

precedence rule + associative rule

Token	Operator	Precedence[1]	Associativity
( ) [ ] -> .	function call array element struct or union member	17	left-to-right
-- ++	increment, decrement[2]	16	left-to-right
-- ++ ! - - + & * sizeof	decrement, increment[3] logical not one's complement unary minus or plus address or indirection size (in bytes)	15	right-to-left
(type)	type cast	14	right-to-left
* / %	mutiplicative	13	Left-to-right

<table>
<tr><td>+ -</td><td>binary add or subtract</td><td>12</td><td>left-to-right</td></tr>
<tr><td><< >></td><td>shift</td><td>11</td><td>left-to-right</td></tr>
<tr><td>> >=
< <=</td><td>relational</td><td>10</td><td>left-to-right</td></tr>
<tr><td>== !=</td><td>equality</td><td>9</td><td>left-to-right</td></tr>
<tr><td>&</td><td>bitwise and</td><td>8</td><td>left-to-right</td></tr>
<tr><td>^</td><td>bitwise exclusive or</td><td>7</td><td>left-to-right</td></tr>
<tr><td>|</td><td>bitwise or</td><td>6</td><td>left-to-right</td></tr>
<tr><td>&&</td><td>logical and</td><td>5</td><td>left-to-right</td></tr>
<tr><td>XX</td><td>logical or</td><td>4</td><td>left-to-right</td></tr>
</table>

<table>
<tr><td>?:</td><td>conditional</td><td>3</td><td>right-to-left</td></tr>
<tr><td>= += -=
/= *= %=
<<= >>=
&= ^= |=</td><td>assignment</td><td>2</td><td>right-to-left</td></tr>
<tr><td>,</td><td>comma</td><td>1</td><td>left-to-right</td></tr>
</table>

1.The precedence column is taken from Harbison and Steele.
2.Postfix form
3.prefix form

Figure 3.12: Precedence hierarchy for C

user **compiler**

Infix	Postfix
2+3*4 a*b+5 (1+2)*7 a*b/c (a/(b-c+d))*(e-a)*c a/b-c+d*e-a*c	234*+ ab*5+ 12+7* ab*c/ abc-d+/ea-*c* ab/c-de*+ac*-

Figure 3.13: Infix and postfix notation

Postfix: no parentheses, no precedence

Token	Stack [0]	[1]	[2]	Top
6	6			0
2	6	2		1
/	6/2			0
3	6/2	3		1
-	6/2-3			0
4	6/2-3	4		1
2	6/2-3	4	2	2
*	6/2-3	4*2		1
+	6/2-3+4*2			0

Figure 3.14: Postfix evaluation

Goal: infix --> postfix

Assumptions:

operators: +, -, *, /, %

operands: single digit integer

```
#define MAX_STACK_SIZE 100 /* maximum stack size */
#define MAX_EXPR_SIZE 100 /* max size of expression */
typedef enum{1paran, rparen, plus, minus, times, divide,
                    mod, eos, operand} precedence;
int stack[MAX_STACK_SIZE]; /* global stack */
char expr[MAX_EXPR_SIZE]; /* input string */
```


eos : end of string

```
int eval(void)
{
/* evaluate a postfix expression, expr, maintained as a
   global variable, '\0' is the the end of the expression.
   The stack and top of the stack are global variables.
   get_token is used to return the token type and
   the character symbol. Operands are assumed to be single
   character digits */
  precedence token;
  char symbol;
  int op1, op2;
  int n = 0;  /* counter for the expression string */
  int top = -1;
  token = get_token(&symbol, &n);
  while (token != eos)
   {
     if (token == operand)
         push(symbol-'0');   /* stack insert */
                             /*  ASCII 0*/
```

exp: character array

```
else {
        /* remove two operands, perform operation, and
           return result to the stack */
    op2 = pop();  /* stack delete */
    op1 = pop();
    switch(token) {
        case plus: add(&top, op1+op2); break;
        case minus: add(&top, op1-op2); break;
        case times: add(&top, op1*op2); break;
        case divide: add(&top, op1/op2); break;
        case mod: add(&top, op1%op2);
        }
      }
  token = get_token (&symbol, &n);
 }
 return pop(); /* return result */
}
```

Program 3.13: Function to evaluate a postfix expression

```
precedence get_token(char *symbol, int *n)
{
/* get the next token, symbol is the character
   representation, which is returned, the token is
   represented by its enumerated value, which
   is returned in the function name */

  *symbol =expr[(*n)++];
  switch (*symbol)  {
    case '(' : return lparen;
    case ')' : return rparen;
    case '+': return plus;
    case '-' : return minus;
```

```
    case '/' :  return divide;
    case '*' : return times;
    case '%' : return mod;
    case '\0' : return eos;
    default  : return operand;
                /* no error checking, default is operand */
    }
}
```

Program 3.14: Function to get a token from the input string

Infix to Postfix Conversion
(Intuitive Algorithm)

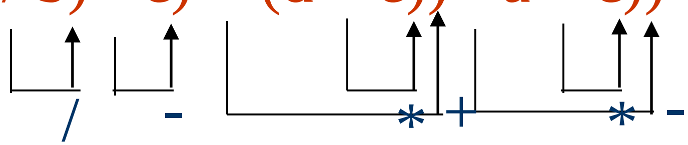
(1) Fully parenthesize expression

a / b - c + d * e - a * c -->

((((a / b) - c) + (d * e)) - a * c))

(2) All operators replace their corresponding right parentheses.

((((a / b) - c) + (d * e)) - a * c))



(3) Delete all parentheses.

ab/c-de*+ac*-

two passes

The orders of operands in infix and postfix are the same.

a + b * c

Token	Stack [0]	Stack [1]	Stack [2]	Top	Output
a				-1	a
+	+			0	a
b	+			0	ab
*	+	*		1	ab
c	+	*		1	abc
eos				-1	abc*+

Figure 3.15: Translation of a+b*c to postfix

$a *_1 (b + c) *_2 d$

Token	Stack [0]	Stack [1]	Stack [2]	Top	Output
a				-1	a
$*_1$	$*_1$			0	a
(	$*_1$	(		1	a
b	$*_1$	(		1	ab
+	$*_1$	(	+	2	ab
c	$*_1$	(	+	2	abc
)	$*_1$	match)		0	abc+
$*_2$	$*_2$	$*_1 = *_2$		0	abc+$*_1$
d	$*_2$			0	abc+$*_1$d
eos	$*_2$			0	abc+$*_1$d$*_2$

Figure 3.16: Translation of a*(b+c)*d to postfix

Rules

(1) Operators are **taken out** of the stack as long as their in-stack precedence (isp) is **higher than or equal to** the incoming precedence (icp) of the new operator.

(2) “(_“ has **low** in-stack precedence, and **high** incoming precedence.

	(	)	+	-	*	/	%	eos
isp	0	19	12	12	13	13	13	0
icp	20	19	12	12	13	13	13	0

```
precedence stack[MAX_STACK_SIZE];
/* isp and icp arrays -- index is value of precedence
lparen, rparen, plus, minus, times, divide, mod, eos */
static int isp [ ] = {0, 19, 12, 12, 13, 13, 13, 0};
static int icp [ ] = {20, 19, 12, 12, 13, 13, 13, 0};
```

isp: in-stack precedence
icp: incoming precedence

```
void postfix(void)
{
/* output the postfix of the expression. The expression
    string, the stack, and top are global */
   char symbol;
   precedence token;
   int n = 0;
   int top = 0; /* place eos on stack */
   stack[0] = eos;
   for (token = get _token(&symbol, &n); token != eos;
                        token = get_token(&symbol, &n)) {
     if (token == operand)
        printf (“%c”, symbol);
     else if (token == rparen ){
```

```
    /*unstack tokens until left parenthesis */
    while (stack[top] != lparen)
        print_token(pop());
    pop(); /*discard the left parenthesis */
   }
   else{
    /* remove and print symbols whose isp is greater
        than or equal to the current token's icp */
    while(isp[stack[top]] >= icp[token] )
        print_token(pop(&top));
    push(token);
   }
 }
 while ((token = pop(&top)) != eos)
     print_token(token);
 print("\n");
}
```

$\theta(n)$

f(n)=θ(g(n)) iff there exist positive constants c_1, c_2, and n_0 such that $c_1 g(n) \le f(n) \le c_2 g(n)$ for all n, $n \ge n_0$.

f(n)=θ(g(n)) iff g(n) is both an upper and lower bound on f(n).

***Program 3.15**: Function to convert from infix to postfix

Infix	Prefix
a*b/c a/b-c+d*e-a*c a*(b+c)/d-g	/*abc -+-/abc*de*ac -/*a+bcdg

(1) evaluation
(2) transformation

***Figure 3.17:** Infix and postfix expressions

Multiple Stacks and Queues

Two stacks

m[0],   m[1],   …,   m[n-2],   m[n-1]

bottommost stack 1 → ← bottommost stack 2

More than two stacks (n)

memory is divided into *n* equal segments

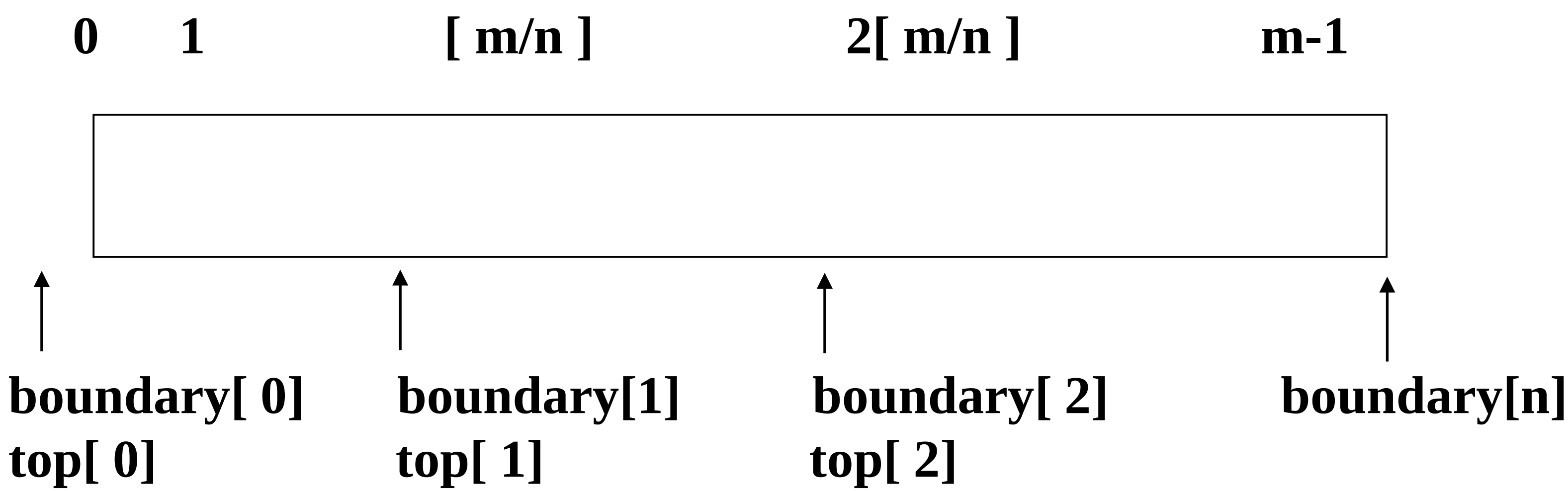
boundary[stack_no]

$0 \le$ stack_no $<$ MAX_STACKS

top[stack_no]

$0 \le$ stack_no $<$ MAX_STACKS

Initially, boundary[i]=top[i].



All stacks are empty and divided into roughly equal segments.

***Figure 3.18:** Initial configuration for *n* stacks in memory [m].

```
#define MEMORY_SIZE 100    /* size of memory */
#define MAX_STACK_SIZE 100
            /* max number of stacks plus 1 */
/* global memory declaration */
element memory[MEMORY_SIZE];
int top[MAX_STACKS];
int boundary[MAX_STACKS];
int n;  /* number of stacks entered by the user */
```

```
top[0] = boundary[0] = -1;
for (i = 1; i < n; i++)
  top[i] =boundary[i] =(MEMORY_SIZE/n)*i;
boundary[n] = MEMORY_SIZE-1;
```

```
void push(int i, element item)
{
   /* add an item to the ith stack */
   if (top[i] == boundary [i+1])
      stackFull(i);
    memory[++top[i]] = item;
}
```

but it may have unused storage

*__Program 3.16:__Add an item to the stack *stack-no*

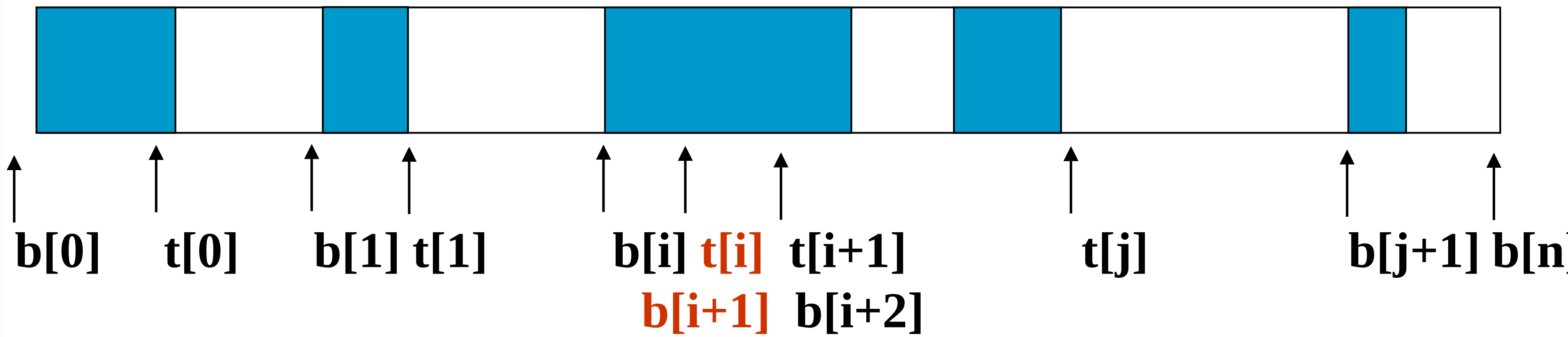
```
element pop(int i)
{
   /* remove top element from the ith stack */
   if (top[i] == boundary[i])
      return stackEmpty(i);
   return memory[top[i]--];
}
```

***Program 3.17:**Delete an *item* from the stack *stack-no*

Find j, stack_no < j < n （往右）

such that top[j] < boundary[j+1]

or, 0 ≤ j < stack_no （往左）



meet

b=boundary, t=top

← 往左或右找一個空間 →

***Figure 3.19:** Configuration when stack i meets stack i+1, but the memory is not full (p.130)