CHAPTER 2

ARRAYS AND STRUCTURES

All the programs in this file are selected from

Ellis Horowitz, Sartaj Sahni, and Susan Anderson-Freed
"Fundamentals of Data Structures in C",

Arrays

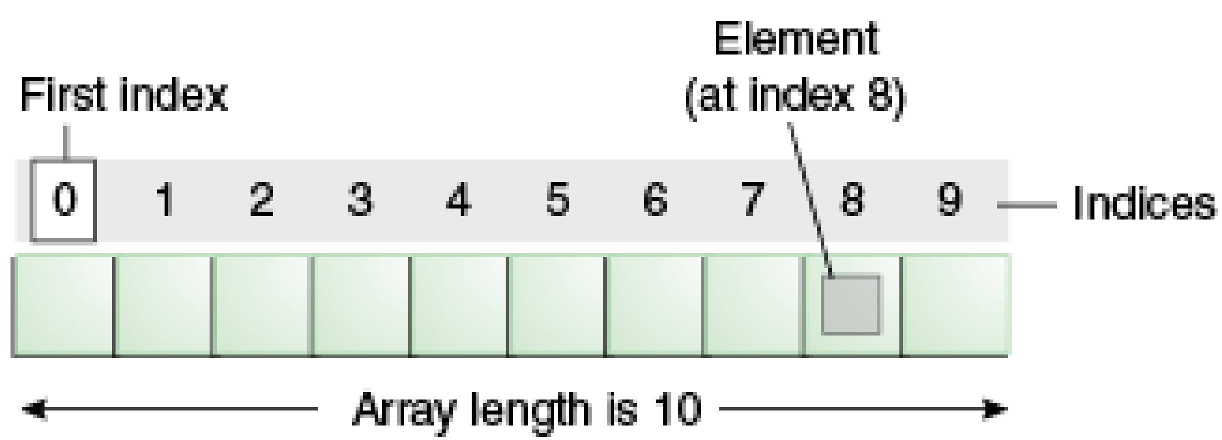
Array: a set of **index** and **value**

data structure:

For each index, there is a value associated with that index.

representation (possible):

implemented by <u>using consecutive memory</u>.



Structure *Array* is

objects: A set of pairs <*index*, *value*> where for each value of *index* there is a value from the set *item*. *Index* is a finite ordered set of one or more dimensions, for example, {0, … , n-1} for one dimension, {(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)} for two dimensions, etc.

Functions:

for all A ∈ Array, *i* ∈ *index*, x ∈ *item*, *j*, *size* ∈ integer

Array Create(j, list) ::= **return** an array of *j* dimensions where list is a j-tuple whose *i*th element is the size of the *i*th dimension. *Items* are undefined.

Item Retrieve(A, *i*) ::= **if** (*i* ∈ *index*) **return** the item associated with index value *i* in array A
else return error

Array Store(A, *i*, *x*) ::= **if (***i* in *index*)
return an array that is identical to array A except the new pair <*i*, *x*> has been inserted **else return** error

end array

***Structure 2.1:** Abstract Data Type *Array*

Arrays in C

int list[5], *plist[5];

list[5]: five integers
list[0], list[1], list[2], list[3], list[4]
*plist[5]: five pointers to integers
plist[0], plist[1], plist[2], plist[3], plist[4]

implementation of 1-D array

list[0]	base address = α
list[1]	α + sizeof(int)
list[2]	α + 2*sizeof(int)
list[3]	α + 3*sizeof(int)
list[4]	α + 4*size(int)

Arrays in C *(Continued)*

Compare int *list1 and int list2[5] in C.

Same: list1 and list2 are **pointers**.
Difference: list2 reserves **five locations**.

Notations:

list2 → a pointer to list2[0]
(list2 + i) → a pointer to list2[i] (&list2[i])
*(list2 + i) → list2[i] (value)

Example: 1-dimension array addressing

int one[] = {0, 1, 2, 3, 4};

Goal: print out address and value

```
void print1(int *ptr, int rows)
{
/* print out a one-dimensional array using a pointer */
        int i;
        printf("Address Contents\n");
        for (i=0; i < rows; i++)
                printf("%8u%5d\n", ptr+i, *(ptr+i));
        printf("\n");
}
```

call print1(&one[0], 5)

Address	Contents
12344868	0
12344872	1
12344876	2
12344880	3
12344884	4

*__Figure 2.1:__ One- dimensional array addressing

Multiple Dimension Array

- Two dimension
  - int arr[2][3];
- Three dimension
  - int arr[2][3][4];
- N dimension
  - int arr[2][3][4][…];

Multidimensional Arrays

C also allows an array to have more than one dimension.

For example, a two-dimensional array consists of a certain number of rows and columns:

```
const int NUMROWS = 3;
const int NUMCOLS = 7;
int Array[NUMROWS][NUMCOLS];
```

	0	1	2	3	4	5	6
0	4	18	9	3	-4	6	0
1	12	45	74	15	0	98	0
2	84	87	75	67	81	85	79

`Array[2][5]` 3rd value in 6th column

`Array[0][4]` 1st value in 5th column

The declaration must specify the number of rows and the number of columns, and both must be constants.

Processing a 2-D Array

A one-dimensional array is usually processed via a for loop.

Similarly, a two-dimensional array may be processed with a nested for loop:

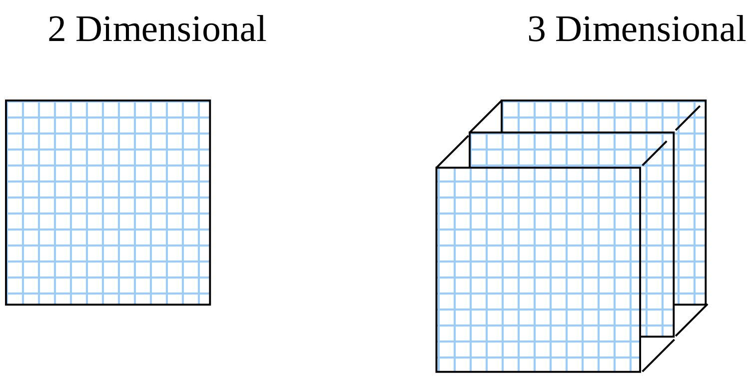
```
for (int Row = 0; Row < NUMROWS; Row++) {
    for (int Col = 0; Col < NUMCOLS; Col++) {
       Array[Row][Col] = 0;
    }
}
```

Each pass through the inner for loop will initialize all the elements of the current row to 0.

The outer for loop drives the inner loop to process each of the array's rows.

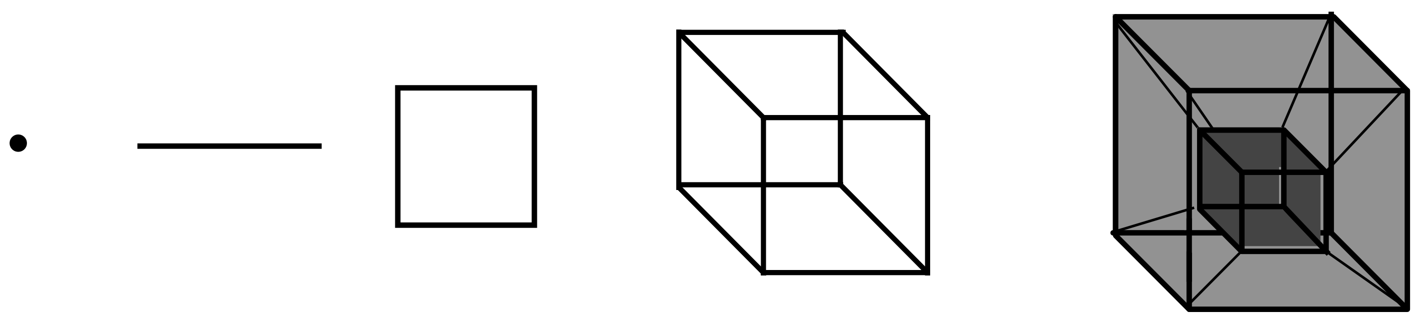
Higher-Dimensional Arrays

An array can be declared with multiple dimensions.




```
double Coord[100][100][100];
```

Multiple dimensions get difficult to visualize graphically.



Structures (records)

```
struct {
        char name[10];
        int age;
        float salary;
        } person;

strcpy(person.name, "james");
person.age=10;
person.salary=35000;
```

Create structure data type

```
typedef struct human_being {
        char name[10];
        int age;
        float salary;
        };

or

typedef struct {
        char name[10];
        int age;
        float salary
        } human_being;

human_being person1, person2;
```

Unions

Example: Add fields for male and female.

```
typedef struct sex_type {
        enum tag_field {female, male} sex;
        union {
                int children;
                int beard;
                } u;
        };

typedef struct human_being {
        char name[10];
        int age;
        float salary;
        date dob;
        sex_type sex_info;
        }
```

Similar to struct, but only one field is active.

```
human_being person1, person2;
person1.sex_info.sex=male;
person1.sex_info.u.beard=0 (False);
```

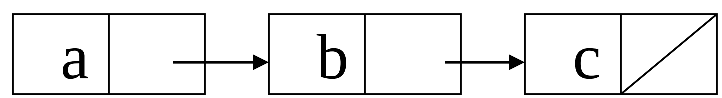
Self-Referential Structures

One or more of its components is a pointer to itself.

```
typedef struct list {
        char data;
        list *link;
        }
```

Construct a list with three nodes
item1.link=&item2;
item2.link=&item3;
malloc: obtain a node

```
list item1, item2, item3;
item1.data='a';
item2.data='b';
item3.data='c';
item1.link=item2.link=item3.link=NULL;
```



Ordered List Examples

ordered (linear) list: (item1, item2, item3, …, item*n*)

- (MONDAY, TUEDSAY, WEDNESDAY, THURSDAY, FRIDAY, SATURDAYY, SUNDAY)
- (2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace)
- (1941, 1942, 1943, 1944, 1945)
- (a_1, a_2, a_3, …, a_{n-1}, a_n)

Operations on Ordered List

1. Find the length, n , of the list.
2. Read the items from left to right (or right to left).
3. Retrieve the i’th element.
4. Store a new value into the i’th position.
5. Insert a new element at the position i , causing elements numbered i, i+1, …, n to become numbered i+1, i+2, …, n+1
6. Delete the element at position i , causing elements numbered i+1, …, n to become numbered i, i+1, …, n-1

array (sequential mapping)? (1)~(4) O (5)~(6) X

Polynomials $A(X)=3X^{20}+2X^5+4$, $B(X)=X^4+10X^3+3X^2+1$

Structure *Polynomial* is

objects: $p(x) = a_1 x^{e_1} + \ldots + a_n x^{e_n}$; a set of ordered pairs of $<e_i,a_i>$ where ***a*_*i*_ in *Coefficients* and *e*_*i*_ in *Exponents***, e_i are integers >= 0

functions:

for all *poly, poly1, poly2* ∈ *Polynomial*, *coef* ∈ *Coefficients*, *expon* ∈ *Exponents*

Polynomial Zero( )	::= **return** the polynomial, $p(x) = 0$
Boolean IsZero(*poly*)	::= **if** (*poly*) **return** *FALSE* **else return** *TRUE*
Coefficient Coef(*poly, expon*)	::= **if** (*expon* ∈ *poly*) **return** its coefficient **else return** Zero
Exponent Lead_Exp(*poly*)	::= **return** the largest exponent in *poly*
Polynomial Attach(*poly,coef, expon*)	::= **if** (*expon* ∈ *poly*) **return** error **else return** the polynomial poly with the term <*coef, expon*> inserted

Polynomial Remove(*poly*, *expon*) ::= **if** (*expon* $\in$ *poly*) **return** the polynomial *poly* with the term whose exponent is *expon deleted* **else return** error

Polynomial SingleMult(*poly*, *coef*, *expon*) ::= **return** the polynomial *poly* • *coef* • x^{expon}

Polynomial Add(*poly1*, *poly2*) ::= **return** the polynomial *poly*1 +*poly*2

Polynomial Mult(*poly1*, *poly2*) ::= **return** the polynomial *poly*1 • *poly*2

End *Polynomial*

***Structure 2.2:**Abstract data type *Polynomial*

Polynomial Addition

data structure 1:

```
#define MAX_DEGREE 101
typedef struct {
        int degree;
        float coef[MAX_DEGREE];
        } polynomial;
```

```
/* d =a + b, where a, b, and d are polynomials */
d = Zero( )
while (! IsZero(a) && ! IsZero(b)) do {
   switch COMPARE (Lead_Exp(a), Lead_Exp(b))  {
      case -1: d =              /* a < b */
         Attach(d, Coef (b, Lead_Exp(b)), Lead_Exp(b));
         b = Remove(b, Lead_Exp(b));
         break;
     case  0: sum = Coef (a, Lead_Exp (a)) + Coef ( b, Lead_Exp(b));
        if (sum) {
           Attach (d, sum, Lead_Exp(a));
           a = Remove(a , Lead_Exp(a));
           b = Remove(b , Lead_Exp(b));
           }
         break;
```

```
case 1: d =
         Attach(d, Coef (a, Lead_Exp(a)), Lead_Exp(a));
         a = Remove(a, Lead_Exp(a));
      }
  }
insert any remaining terms of a or b into d
```

advantage: easy implementation
disadvantage: waste space when sparse

***Program 2.5 :**Initial version of *padd* function

Data structure 2: use **<u>one global array</u>** to store all polynomials

$A(X)=2X^{1000}+1$

$B(X)=X^4+10X^3+3X^2+1$

starta *finisha* *startb* *finishb* *avail*

coef	2	1	1	10	3	1	
exp	1000	0	4	3	2	0	
	0	1	2	3	4	5	6

specification	representation
poly	<start, finish>
A	<0,1>
B	<2,5>

```
MAX_TERMS 100 /* size of terms array */
typedef struct {
        float coef;
        int expon;
        } polynomial;
polynomial terms[MAX_TERMS];
int avail = 0;
```

Add two polynomials: D = A + B

```
void padd (int starta, int finisha, int startb, int finishb,
                                              int * startd, int *finishd)
{
/* add A(x) and B(x) to obtain D(x) */
    float coefficient;
   *startd = avail;
   while (starta <= finisha && startb <= finishb)
     switch (COMPARE(terms[starta].expon,
                                    terms[startb].expon)) {
      case -1: /* a expon < b expon */
              attach(terms[startb].coef, terms[startb].expon);
              startb++
              break;
```

```
case  0: /* equal exponents */
          coefficient = terms[starta].coef +
                        terms[startb].coef;
          if (coefficient)
            attach (coefficient, terms[starta].expon);
          starta++;
          startb++;
          break;
case 1: /* a expon > b expon */
      attach(terms[starta].coef, terms[starta].expon);
      starta++;
}
```

```
/* add in remaining terms of  A(x) */
for( ; starta <= finisha; starta++)
     attach(terms[starta].coef, terms[starta].expon);
/* add in remaining terms of B(x) */
for( ; startb <= finishb; startb++)
    attach(terms[startb].coef, terms[startb].expon);
*finishd =avail -1;
}
```

Analysis: $O(n+m)$

where n (m) is the number of nonzeros in A(B).

***Program 2.6:** Function to add two polynomial

```
void attach(float coefficient, int exponent)
{
/* add a new term to the polynomial */
    if (avail >= MAX_TERMS) {
       fprintf(stderr, "Too many terms in the polynomial\n");
       exit(1);
      }
      terms[avail].coef  = coefficient;
      terms[avail++].expon = exponent;
}
```

Problem: Compaction is required when polynomials that are no longer needed. (data movement takes time.)

Sparse Matrix

$$
\begin{array}{c|ccc|}
 & \text{col 1} & \text{col 2} & \text{col 3} \\
\text{row 1} & -27 & 3 & 4 \\
\text{row 2} & 6 & 82 & -2 \\
\text{row 3} & 109 & -64 & 11 \\
\text{row 4} & 12 & 8 & 9 \\
\text{row 5} & 48 & 27 & 47
\end{array}
$$

5*3

$$
\begin{array}{c|cccccc|}
 & \text{col1} & \text{col2} & \text{col3} & \text{col4} & \text{col5} & \text{col6} \\
\text{row0} & 15 & 0 & 0 & 22 & 0 & -15 \\
\text{row1} & 0 & 11 & 3 & 0 & 0 & 0 \\
\text{row2} & 0 & 0 & 0 & -6 & 0 & 0 \\
\text{row3} & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{row4} & 91 & 0 & 0 & 0 & 0 & 0 \\
\text{row5} & 0 & 0 & 28 & 0 & 0 & 0
\end{array}
$$

6*6

(a) 15/15

(b) 8/36

<u>Figure 2.4: Two matrices</u>

sparse matrix
data structure?

SPARSE MATRIX ABSTRACT DATA TYPE

Structure *Sparse_Matrix* is

objects: a set of triples, <*row, column, value*>, where *row* and *column* are integers and form a unique combination, and *value* comes from the set *item*.

functions:

for all $a, b \in Sparse\_Matrix$, $x \in item$, $i, j, max\_col, max\_row \in index$

Sparse_Marix Create(*max_row*, *max_col*) ::=

return a *Sparse_matrix* that can hold up to $max\_items = max\_row \times max\_col$ and whose maximum row size is *max_row* and whose maximum column size is *max_col*.

Sparse_Matrix Transpose(*a*) ::=

return the matrix produced by interchanging the row and column value of every triple.

Sparse_Matrix Add(*a*, *b*) ::=

if the dimensions of a and b are the same

return the matrix produced by adding corresponding items, namely those with identical *row* and *column* values.

else return error

Sparse_Matrix Multiply(*a*, *b*) ::=

if number of columns in **a** equals number of rows in **b**

return the matrix *d* produced by multiplying a by *b* according to the formula: $d[i][j] = \Sigma(a[i][k] \cdot b[k][j])$ where $d(i, j)$ is the (i,j)th element

else return error.

* **Structure 2.3:** Abstract data type Sparse-Matrix

(1) Represented by a two-dimensional array.
Sparse matrix wastes space.

(2) Each element is characterized by <row, col, value>.

# of rows (columns)

# of nonzero terms

	row	col	value
a[0]	6	6	8
[1]	0	0	15
[2]	0	3	22
[3]	0	5	-15
[4]	1	1	11
[5]	1	2	3
[6]	2	3	-6
[7]	4	0	91
[8]	5	2	28

(a)

transpose

	row	col	value
b[0]	6	6	8
[1]	0	0	15
[2]	0	4	91
[3]	1	1	11
[4]	2	1	3
[5]	2	5	28
[6]	3	0	22
[7]	3	2	-6
[8]	5	0	-15

(b)

row, column in ascending order

Figure 2.5: Sparse matrix and **its transpose** stored as triples

Sparse_matrix Create(max_row, max_col) ::=

```
#define MAX_TERMS 101 /* maximum number of terms +1*/
    typedef struct {
            int col;
            int row;
            int value;
            } term;
    term a [MAX_TERMS]
```

# of rows
# of columns
# of nonzero terms

Transpose a Matrix

(1) for each row i

take element <i, j, value> and store it
in element <j, i, value> of the transpose.

difficulty: where to put <j, i, value>

(0, 0, 15) ====> (0, 0, 15)
(0, 3, 22) ====> (3, 0, 22)
(0, 5, -15) ====> (5, 0, -15)
(1, 1, 11) ====> (1, 1, 11)

Move elements down very often.

(2) For all elements in column j,

place element <i, j, value> in element <j, i, value>

```
void transpose (term a[], term b[])
/* b is set to the transpose of a */
{
   int n, i, j, currentb;
   n = a[0].value;  /* total number of elements */
   b[0].row = a[0].col;  /* rows in b = columns in a */
   b[0].col = a[0].row;  /*columns in b = rows in a */
   b[0].value = n;
   if (n > 0) {                /*non zero matrix */
      currentb = 1;
      for (i = 0; i < a[0].col; i++)
      /* transpose by columns in a */
         for( j = 1; j <=  n; j++)
         /*  find elements from the current column */
         if (a[j].col == i) {
        /* element is in current column, add it to b */
```

columns

elements

```
b[currentb].row = a[j].col;
b[currentb].col  = a[j].row;
b[currentb].value = a[j].value;
currentb++
}
}
}
```

*** Program 2.8:** Transpose of a sparse matrix

Scan the array "**columns**" times.
The array has "**elements**" elements.

==> O(columns*elements)

Discussion: compared with 2-D array representation

O(columns*elements) vs. O(columns*rows)

elements --> columns * rows when nonsparse
O(columns*columns*rows)

Problem: **Scan the array "columns^2*rows" times**.

Solution:

Determine the number of elements in each column of the original matrix.
==>
Determine the starting positions of each row in the transpose matrix.

```
void fast_transpose(term a[ ], term b[ ])
{
/* the transpose of a is placed in b */
  int row_terms[MAX_COL], starting_pos[MAX_COL];
  int i, j, num_cols = a[0].col, num_terms = a[0].value;
  b[0].row = num_cols; b[0].col = a[0].row;
  b[0].value = num_terms;
  if (num_terms > 0){ /*nonzero matrix*/
             for (i = 0; i < num_cols; i++)
columns            row_terms[i] = 0;
             for (i = 1; i  <= num_terms; i++) /*計算 row_terms的值*
elements           row_term [a[i].col]++
             starting_pos[0] = 1;
             for (i =1; i < num_cols; i++)
columns            starting_pos[i]=starting_pos[i-1] +row_terms [i-1];
  /*計算 starting_pos的值*/
```

elements

```
        for (i=1; i <= num_terms, i++) {
                j = starting_pos[a[i].col]++;
                b[j].row = a[i].col;
                b[j].col = a[i].row;
                b[j].value = a[i].value;
        }
    }
}
```

***Program 2.9:**Fast transpose of a sparse matrix

Compared with 2-D array representation

O(columns+**elements**) vs. O(columns*rows)

elements --> columns * rows

O(columns+elements) --> O(columns*rows)

Cost: Additional row_terms and starting_pos arrays are required.

Let the two arrays row_terms and starting_pos be shared.

a[0]	6	6	8
a[1]	0	0	15
a[2]	0	3	22
a[3]	0	5	-15
a[4]	1	1	11
a[5]	1	2	3
a[6]	2	3	-6
a[7]	4	0	91
a[8]	5	2	28

starting_pos[i]=
starting_pos[i-1] +row_terms [

	[0]	[1]	[2]	[3]	[4]	[5]
row_terms =	2	1	2	2	0	1
starting_pos =	1	3	4	6	8	8

Compare

	space	time
2D array	O(rows * cols)	O(rows * cols)
Transpose	O(elements)	O(cols * elements)
Fast Transpose	O(elements+MAX_COL)	O(cols + elements)

Sparse Matrix Multiplication

Definition: $[D]_{m*p}=[A]_{m*n}*[B]_{n*p}$

Procedure: Fix a row of A and find all elements in column j of B for j=0, 1, …, p-1.

Alternative 1. Scan all of B to find all elements in j.

Alternative 2. Compute the transpose of B.
(Put all column elements consecutively)

$d_{ij}=a_{i0}*b_{0j}+a_{i1}*b_{1j}+\ldots+a_{i(n-1)*}b_{(n-1)j}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

***Figure 2.5:**Multiplication of two sparse matrices

```
void mmult (term a[ ], term b[ ], term d[ ] )
/* multiply two sparse matrices */
{
   int i, j, column, totalb = b[].value, totald = 0;
   int rows_a = a[0].row, cols_a = a[0].col,
   totala = a[0].value; int cols_b = b[0].col,
   int row_begin = 1, row = a[1].row, sum =0;
   int new_b[MAX_TERMS][3];
   if (cols_a != b[0].row){
   /*compare the row of a and the col of b*/
         fprintf (stderr, “Incompatible matrices\n”);
         exit (1);
}
```

```
fast_transpose(b, new_b); /* the transpose of b is placed in new_b */
/* set boundary condition */  cols_b + totalb
a[totala+1].row = rows_a; /* a[0].row*/
new_b[totalb+1].row = cols_b;
new_b[totalb+1].col = 0;
for (i = 1; i <= totala; ) {/* a[0].val* /      at most rows_a times
   column = new_b[1].row;  /* b[1].col*/
   for (j = 1; j <= totalb+1;) {  /*b[0].val*/  at most cols_b times
   /* mutiply row of a by column of b */
   if (a[i].row != row)  {  /* a[1].row */
      storesum(d, &totald, row, column, &sum);
      i = row_begin;
      for (; new_b[j].row == column; j++)
        ;
      column =new_b[j].row;
   }
```

```
        else switch (COMPARE (a[i].col, new_b[j].col)) {
            case -1: /* go to next term in a */
                     i++; break;
            case 0: /* add terms, go to next term in a and b */
                     sum += (a[i++].value * new_b[j++].value);
                     break;
             case 1: /* advance to next term in b*/
                      j++
          }
    } /* end of for j <= totalb+1 */
    for (; a[i].row == row; i++)
          ;
     row_begin = i; row = a[i].row;
  } /* end of for i <=totala */
  d[0].row = rows_a; /* a[0].row*/
  d[0].col = cols_b; /* b[0].cols*/
  d[0].value = totald;
}
```

```
void storesum(term d[ ], int *totald, int row, int column,
                                             int *sum)
{
/* if *sum != 0, then it along with its row and column
   position is stored as the *totald+1 entry in d */
   if (*sum)
     if (*totald < MAX_TERMS) {
       d[++*totald].row = row;
       d[*totald].col = column;
       d[*totald].value = *sum;
     }
    else {
      fprintf(stderr, "Numbers of terms in product
                                    exceed %d\n", MAX_TERMS);
 exit(1);
 }
}
```


Program 2.11: storsum function

Analyzing the algorithm

$\text{cols\_b} * \text{termsrow}_1 + \text{totalb} +$
$\text{cols\_b} * \text{termsrow}_2 + \text{totalb} +$
$\ldots +$
$\text{cols\_b} * \text{termsrow}_p + \text{totalb}$
$= \text{cols\_b} * (\text{termsrow}_1 + \text{termsrow}_2 + \ldots + \text{termsrow}_p) +$
$\text{rows\_a} * \text{totalb}$
$= \text{cols\_b} * \text{totala} + \text{row\_a} * \text{totalb}$

$O(\text{cols\_b} * \text{totala} + \text{rows\_a} * \text{totalb})$

Compared with matrix multiplication using array

```
for (i =0; i < rows_a; i++)
    for (j=0; j < cols_b; j++) {
        sum =0;
        for (k=0; k < cols_a; k++)
            sum += (a[i][k] *b[k][j]);
        d[i][j] =sum;
    }
```

O(rows_a * cols_a * cols_b) vs.
O(cols_b * total_a + rows_a * total_b)

optimal case: total_a < rows_a * cols_a
total_b < cols_a * cols_b

worse case: total_a --> rows_a * cols_a, or
total_b --> cols_a * cols_b