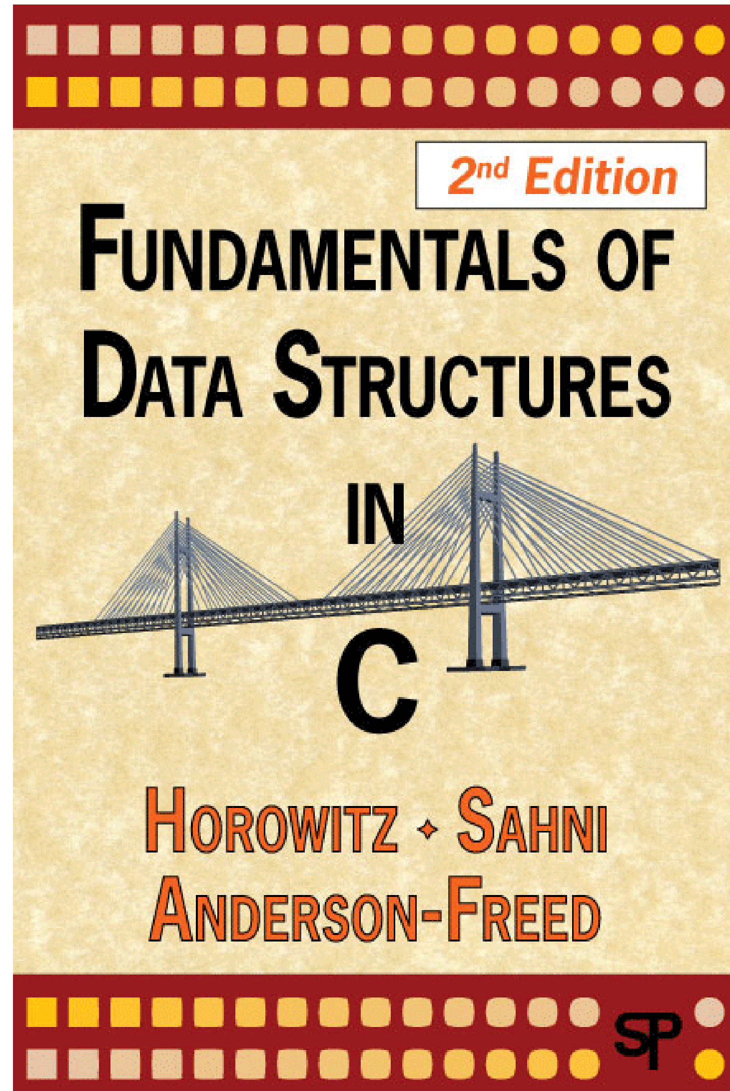
Data Structures

Books

Fundamentals of Data Structures in C, 2nd Edition.

(開發圖書，(02) 8242-3988)



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  - Quiz 20%
  - Computer-based Test 20%
  - Homework 20%
  - Midterm Exam 25%
  - Final Exam 25%

Introductory

- Raise your hand is always welcome!
- No phone, walk, sleep, and late during the lecture time.
- Data structure is not the fundamental course for programming.
- Slides are not enough. To master the materials, page-by-page reading is necessary.

Outline

- Basic Concept
- Arrays and Structures
- Stacks and Queues
- Lists
- Trees
- Graphs
- Sorting
- Hashing

CHAPTER 1

BASIC CONCEPT

All the programs in this file are selected from

Ellis Horowitz, Sartaj Sahni, and Susan Anderson-Freed

“Fundamentals of Data Structures in C”,

How to create programs

- Requirements
- Analysis: bottom-up vs. top-down
- Design: data objects and operations
- Refinement and Coding
- Verification
  - Program Proving
  - Testing
  - Debugging

Algorithm

- Definition
  An *algorithm* is a finite set of instructions that accomplishes a particular task.
- Criteria
  - input
  - output
  - definiteness: clear and unambiguous
  - finiteness: terminate after a finite number of steps
  - effectiveness: instruction is basic enough to be carried out

Data Type

- Data Type
  A *data type* is a collection of *objects* and a set of *operations* that act on those objects.
- Abstract Data Type (ADT)
  An *ADT* is a data type that is organized in such a way that **the specification of the objects and the operations on the objects** is separated from
  - the representation of the objects .
  - the implementation of the operations.

Specification vs. Implementation

- Operation specification
  - function name
  - the types of arguments
  - the type of the results
- Implementation independent

Structure 1.1:** Abstract data type *Natural_Number

structure Natural_Number is

objects: an ordered subrange of the integers starting at zero and ending at the maximum integer (*INT_MAX*) on the computer

functions:

for all x, y ∈ *Nat_Number*; *TRUE*, *FALSE* ∈ *Boolean*

and where +, -, <, and == are the usual integer operations.

Nat_Num Zero ( ) ::= 0

Boolean Is_Zero(x) ::= **if** (x) **return** *FALSE*
else return *TRUE*

Nat_Num Add(x, y) ::= **if** ((x+y) <= *INT_MAX*) **return** x+y
else return *INT_MAX*

Boolean Equal(x,y) ::= **if** (x== y) **return** *TRUE*
else return *FALSE*

Nat_Num Successor(x) ::= **if** (x == *INT_MAX*) **return** x
else return x+1

Nat_Num Subtract(x,y) ::= **if** (x<y) **return** 0
else return x-y

end *Natural_Number*

::= is defined as

Measurements

- Criteria
  - Is it correct?
  - Is it readable?
  - …
- Performance Measurement (machine dependent)
- Performance Analysis (machine independent)
  - space complexity: storage requirement
  - time complexity: computing time

Space Complexity $S(P)=C+S_P(I)$

- Fixed Space Requirements (C)
  Independent of the characteristics of the inputs and outputs
  - instruction space
  - space for simple variables, fixed-size structured variable, constants
- Variable Space Requirements ($S_P(I)$)
  depend on the instance characteristic I
  - number, size, values of inputs and outputs associated with I
  - recursive stack space, formal parameters, local variables, return address

***Program 1.10:** Simple arithmetic function

```
float abc(float a, float b, float c)
{
   return a + b + b * c + (a + b - c) / (a + b) + 4.00;
 }
```

This function has only fixed space requirements $S_{abc}(I) = 0$

***Program 1.11:** Iterative function for summing a list of numbers

```
float sum(float list[ ], int n)
{
 float tempsum = 0;
 int i;
 for (i = 0; i<n; i++)
    tempsum += list [i];
 return tempsum;
}
```

$S_{sum}(I) = 0$

Recall: pass the address of the first element of the array & pass by value

***Program 1.12:** Recursive function for summing a list of numbers

```
float rsum(float list[ ], int n)
{
  if (n) return rsum(list, n-1) + list[n-1];
  return 0;
}
```

$S_{sum}(I)=S_{sum}(n)=12n$

Assumptions:

***Figure 1.1:** Space needed for one recursive call of Program 1.12

Type	Name	Number of bytes
parameter: array pointer parameter: integer return address:(used internally)	list [ ] n	4 4 4 (unless a far address)
TOTAL per recursive call		12

Time Complexity

$$T(P)=C+T_P(I)$$

- Compile time (C)
  independent of instance characteristics
- Run (execution) time T_P
- Definition

  $$T_P(n)=c_a ADD(n)+c_s SUB(n)+c_l LDA(n)+c_{st} STA(n)$$

  A *program step* is a syntactically or semantically meaningful program segment whose execution time is independent of the instance characteristics.
- Example
  – abc = a + b + b * c + (a + b - c) / (a + b) + 4.0
  – abc = a + b + c

Regard as the same unit
machine independent

Methods to compute the step count

- Introduce variable count into programs
- **Tabular** method
  - Determine the total number of steps contributed by each statement
    step per execution × frequency
  - add up the contribution of all statements

Tabular Method

***Figure 1.2:** Step count table for Program 1.11

Iterative function to sum a list of numbers

steps/execution

Statement	s/e	Frequency	Total steps
float sum(float list[ ], int n)	0	0	0
{	0	0	0
float tempsum = 0;	1	1	1
int i;	0	0	0
for(i=0; i <n; i++)	1	n+1	n+1
tempsum += list[i];	1	n	n
return tempsum;	1	1	1
}	0	0	0
Total			2n+3

Iterative summing of a list of numbers

***Program 1.13:** Program 1.11 with count statements

```
float sum(float list[ ], int n)
{
   float tempsum = 0; count++; /* for assignment */
   int i;
   for (i = 0; i < n; i++) {
        count++;           /*for the for loop */
        tempsum += list[i]; count++;  /* for assignment */
   }
   count++;        /* last execution of for */
   count++;        /* for return */
   return tempsum;

}
```

2n + 3 steps

***Program 1.14:** Simplified version of Program 1.13

```
float sum(float list[ ], int n)
{
   float tempsum = 0;
   int i;
   for (i = 0; i < n; i++)
      count += 2;
   count += 3;
   return 0;
}
```

count final value is
2n + 3

Recursive summing of a list of numbers

***Program 1.15:** Program 1.12 with count statements added*

```
float rsum(float list[ ], int n)
{
        count++;      /*for if conditional */
        if (n) {
                count++;  /* for return and rsum invocation */
                return rsum(list, n-1) + list[n-1];
        }
        count++;
        return list[0];
}
```

2n+2

Recursive Function to sum of a list of numbers

***Figure 1.3:** Step count table for recursive summing function

Statement	s/e	Frequency	Total steps
float rsum(float list[ ], int n)	0	0	0
{	0	0	0
if (n)	1	n+1	n+1
return rsum(list, n-1)+list[n-1];	1	n	n
return list[0];	1	1	1
}	0	0	0
Total			2n+2

Matrix addition

***Program 1.16:** Matrix addition

```
void add( int a[ ] [MAX_SIZE], int b[ ] [MAX_SIZE],
                     int c [ ] [MAX_SIZE], int rows, int cols)
{
   int i, j;
   for (i = 0; i < rows; i++)
      for (j= 0; j < cols; j++)
         c[i][j] = a[i][j] +b[i][j];
 }
```

rows * cols

Matrix Addition

***Figure 1.4:** Step count table for matrix addition*

Statement	s/e	Frequency	Total steps
Void add (int a[ ][MAX_SIZE] • • •)	0	0	0
{	0	0	0
int i, j;	0	0	0
for (i = 0; i < row; i++)	1	rows+1	rows+1
for (j=0; j< cols; j++)	1	rows • (cols+1)	rows • cols+rows
c[i][j] = a[i][j] + b[i][j];	1	rows • cols	rows • cols
}	0	0	0
Total			2rows • cols+2rows+1

***Program 1.17:** Matrix addition with count statements

```
void add(int a[ ][MAX_SIZE], int b[ ][MAX_SIZE],
                          int c[ ][MAX_SIZE], int row, int cols )
{
   int i, j;
   for (i = 0; i < rows; i++){
        count++; /* for i for loop */
        for (j = 0; j < cols; j++) {
           count++; /* for j for loop */
           c[i][j] = a[i][j] + b[i][j];
           count++; /* for assignment statement */
        }
        count++;    /* last time of j for loop */
   }
  count++;        /* last time of i for loop */
}
```

2*rows * cols + 2 rows + 1

***Program 1.18:** Simplification of Program 1.17

```
void add(int a[ ][MAX_SIZE], int b [ ][MAX_SIZE],
                    int c[ ][MAX_SIZE], int rows, int cols)
{
   int i, j;
   for( i = 0; i < rows; i++) {
     for (j = 0; j < cols; j++)
        count += 2;
        count += 2;
   }
   count++;
}
```

$2*rows \times cols + 2rows + 1$

Suggestion: Interchange the loops when rows >> cols

Exercise 1

***Program 1.19:** Printing out a matrix*

```
void print_matrix(int matrix[ ][MAX_SIZE], int rows, int cols)
{
   int i, j;
   for (i = 0; i < row; i++) {          /* row +1*/
      for (j = 0; j < cols; j++)        /*  row * (col +1) */
         printf("%d",  matrix[i][j]);   / row * col */
      printf( "\n");                    /* row */
   }
}
```

2*row*col + 2 row + row +1

Exercise 2

***Program 1.20:Matrix multiplication function**

```
void mult(int a[ ][MAX_SIZE], int b[ ][MAX_SIZE], int c[ ][MAX_SIZE])
{
  int i, j, k;
  for (i = 0; i < MAX_SIZE; i++)
    for (j = 0; j< MAX_SIZE;  j++) {
       c[i][j] = 0;
    for (k = 0; k < MAX_SIZE; k++)
        c[i][j]  +=  a[i][k] * b[k][j];
    }
}
```

$O(MAX\_SIZE)^3$

Exercise 3

***Program 1.21:Matrix product function**

```
void prod(int a[ ][MAX_SIZE], int b[ ][MAX_SIZE], int c[ ][MAX_SIZE],
                                                  int rowsa, int colsb, int colsa)
{
  int i, j, k;
  for (i = 0; i < rowsa; i++)     /* rowsa +1*/
    for (j = 0; j< colsb;  j++) { /*  rowsa * (colsb +1) */
      c[i][j] = 0;                /*  rowsa * colsb */
    for (k = 0; k< colsa; k++)    /*  rowsa * colsb * (colsa +1) */
        c[i][j]  +=  a[i][k] * b[k][j]; /*  rowsa * colsb  * colsa */
    }
}
```

???

Exercise 4

***Program 1.22:Matrix transposition function**

```
void transpose(int a[ ][MAX_SIZE])
{
  int i, j, temp;
  for (i = 0; i < MAX_SIZE-1; i++)       /* MAX_SIZE */
    for (j = i+1; j < MAX_SIZE;  j++) /* [(MAX_SIZE +2 ) (MAX_SIZE-1)]/2*/
        SWAP (a[i][j], a[j][i], temp);     /* (MAX_SIZE-1)^2 / 2 */
}
```

???

Asymptotic Notation (O)

- Definition
  $f(n) = O(g(n))$ iff there exist positive constants c and n_0 such that $f(n) \le cg(n)$ for all n, $n \ge n_0$.
- Examples
  - $3n+2=O(n)$ /* $3n+2\le 4n$ for $n\ge 2$ */
  - $3n+3=O(n)$ /* $3n+3\le 4n$ for $n\ge 3$ */
  - $100n+6=O(n)$ /* $100n+6\le 101n$ for $n\ge 6$ */
  - $10n^2+4n+2=O(n^2)$ /* $10n^2+4n+2\le 11n^2$ for $n\ge 5$ */
  - $6*2^n+n^2=O(2^n)$ /* $6*2^n+n^2 \le 7*2^n$ for $n\ge 4$ */

Example

- Complexity of $c_1n^2+c_2n$ and c_3n
  - for sufficiently large of value, c_3n is faster than $c_1n^2+c_2n$
  - for small values of n, either could be faster
    - $c_1=1$, $c_2=2$, $c_3=100$ --> $c_1n^2+c_2n \leq c_3n$ for $n \leq 98$
    - $c_1=1$, $c_2=2$, $c_3=1000$ --> $c_1n^2+c_2n \leq c_3n$ for $n \leq 998$
  - break even point
    - no matter what the values of c1, c2, and c3, the n beyond which c_3n is always faster than $c_1n^2+c_2n$

- O(1): constant
- O(n): linear
- O(n^2): quadratic
- O(n^3): cubic
- O(2^n): exponential
- O(logn)
- O(nlogn)

*Figure 1.7:Function values

		Instance characteristic n					
Time	Name	1	2	4	8	16	32
1	Constant	1	1	1	1	1	1
$\log n$	Logarithmic	0	1	2	3	4	5
n	Linear	1	2	4	8	16	32
$n \log n$	Log linear	0	2	8	24	64	160
n^2	Quadratic	1	4	16	64	256	1024
n^3	Cubic	1	8	64	512	4096	32768
2^n	Exponential	2	4	16	256	65536	4294967296
$n!$	Factorial	1	2	24	40326	20922789888000	26313×10^{33}

Figure 1.7 Function values

***Figure 1.8:** Plot of function values

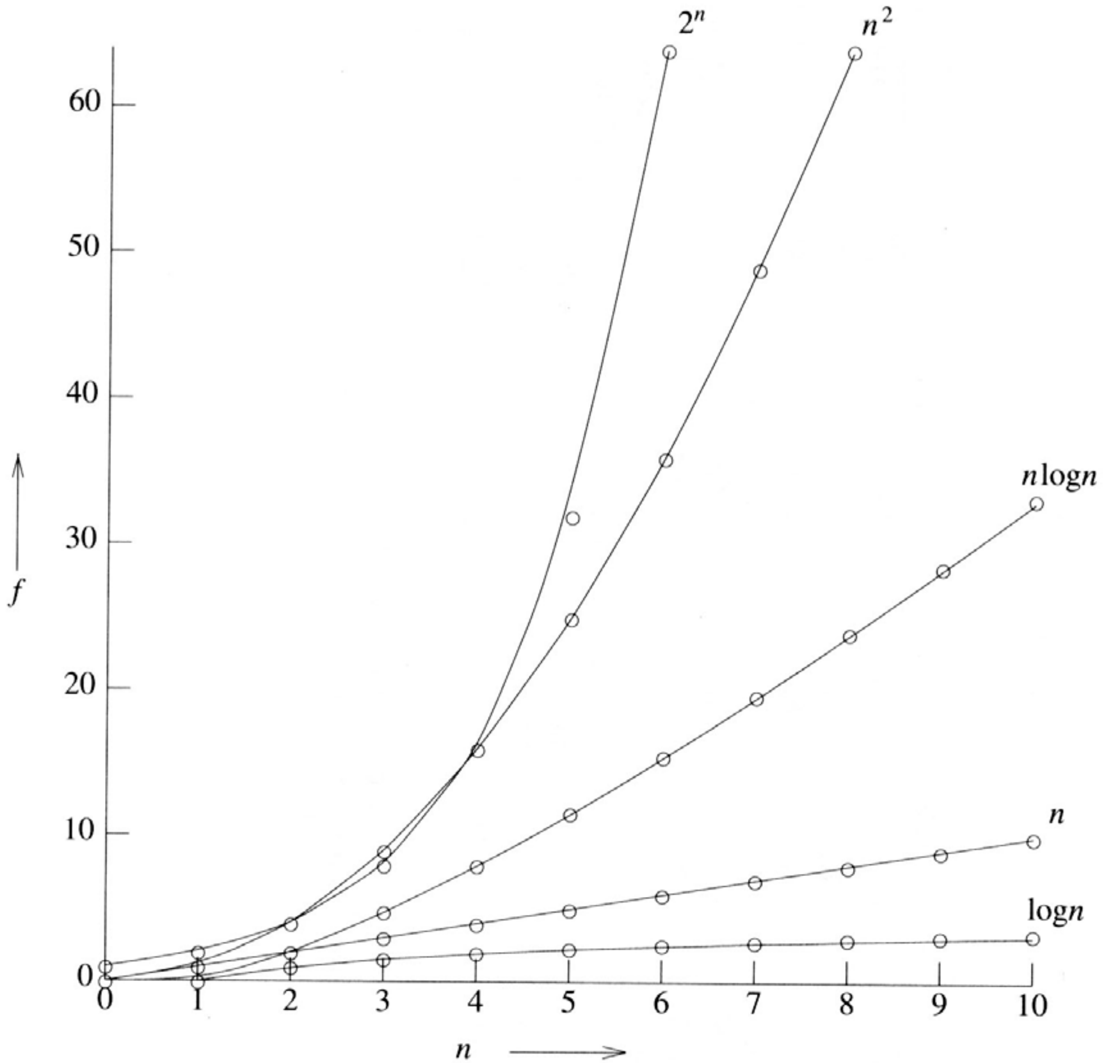

Figure 1.8 Plot of function values

***Figure 1.9:**Times on a 1 billion instruction per second computer

	$f(n)$						
n	n	$n \log_2 n$	n^2	n^3	n^4	n^{10}	2^n
10	.01 µs	.03 µs	.1 µs	1 µs	10 µs	10s	1µs
20	.02 µs	.09 µs	.4 µs	8 µs	160 µs	2.84h	1ms
30	.03 µs	.15 µs	.9 µs	27 µs	810 µs	6.83d	1s
40	.04 µs	.21 µs	1.6 µs	64 µs	2.56ms	121d	18m
50	.05 µs	.28 µs	2.5 µs	125 µs	6.25ms	3.1y	13d
100	.10 µs	.66 µs	10 µs	1ms	100ms	3171y	$4*10^{13}$y
10^3	1 µs	9.96 µs	1 ms	1s	16.67m	$3.17*10^{13}$y	$32*10^{283}$y
10^4	10 µs	130 µs	100 ms	16.67m	115.7d	$3.17*10^{23}$y	
10^5	100 µs	1.66 ms	10s	11.57d	3171y	$3.17*10^{33}$y	
10^6	1ms	19.92ms	16.67m	31.71y	$3.17*10^7$y	$3.17*10^{43}$y	