



## CHAPTER 4

# LISTS

All the programs in this file are selected from

Ellis Horowitz, Sartaj Sahni, and Susan Anderson-Freed  
“Fundamentals of Data Structures in C”,



# Introduction

- Array

- successive items locate a fixed distance

- disadvantage

- data movements during insertion and deletion
  - waste space in storing  $n$  ordered lists of varying size

- possible solution

- Linked List**



# Pointer

pointer

```
int i, *pi;
```

```
pi= (int *) malloc(sizeof(int));
```

```
/* assign to pi a pointer to int */
```

```
pi = &i;
```

```
i=10; *pi=10
```

```
pf=(float *) pi;
```

```
/* coverts an int pointer to a float pointer */
```

# malloc()

- The C library function **void \*malloc(size\_t size)** allocates the requested memory and returns a pointer to it.

```
#include <stdio.h>
#include <stdlib.h>

int main()
{
    char *str;

    /* Initial memory allocation */
    str = (char *) malloc(15);
    strcpy(str, "tutorialspoint");
    printf("String = %s, Address = %u\n", str, str);

    /* Reallocating memory */
    str = (char *) realloc(str, 25);
    strcat(str, ".com");
    printf("String = %s, Address = %u\n", str, str);

    free(str);

    return(0);
}
```

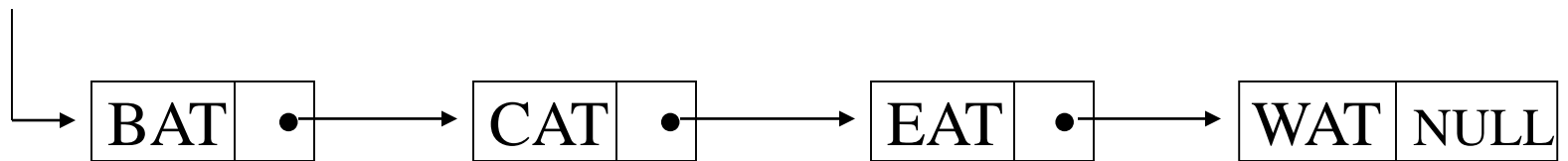
```
String = tutorialspoint, Address = 355090448
```

```
String = tutorialspoint.com, Address = 355090448
```

# Using Dynamically Allocated Storage

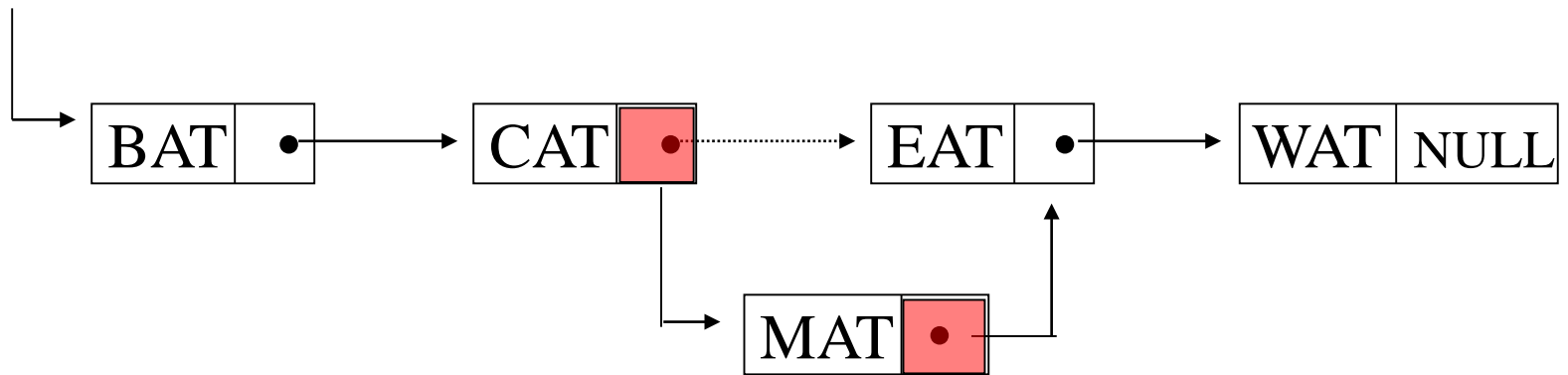
```
int i, *pi;  
float f, *pf;  
pi = (int *) malloc(sizeof(int));    → request memory  
pf = (float *) malloc (sizeof(float));  
*pi =1024;  
*pf =3.14;  
printf("an integer = %d, a float = %f\n", *pi, *pf);  
free(pi);                            → return memory  
free(pf);
```

# Singly Linked Lists



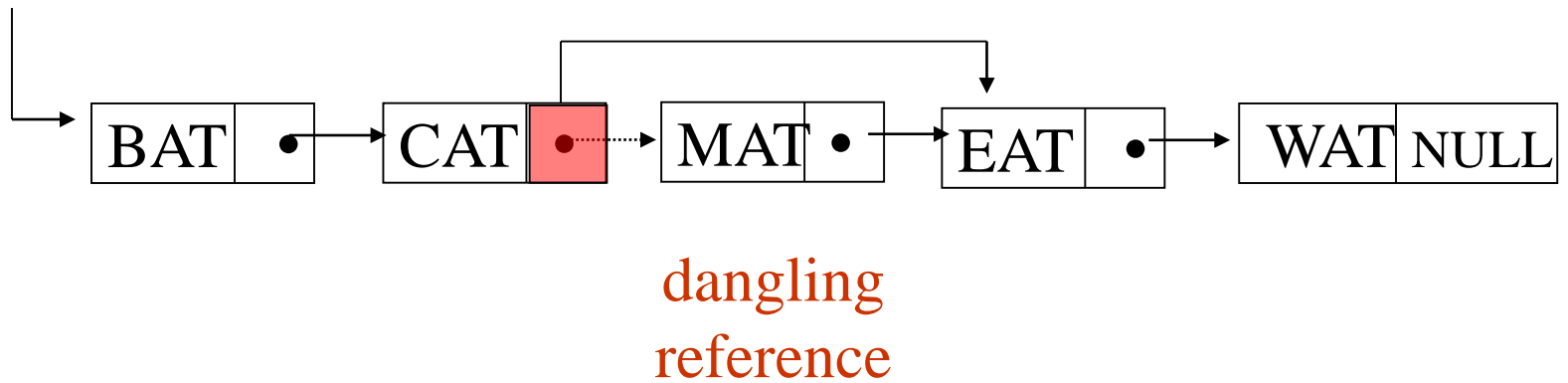
\*Figure 4.2: Usual way to draw a linked list

Insert



**Figure 4.3:** Insert MAT after CAT

# Delete



\*Figure 4.4: Delete MAT from list



# Example 4.1: Create a linked list of words

## Declaration

```
typedef struct list_node, *list_pointer;  
typedef struct list_node {  
    char data [4];  
    list_pointer link;  
};
```

## Creation

```
list_pointer first =NULL;
```

## Testing

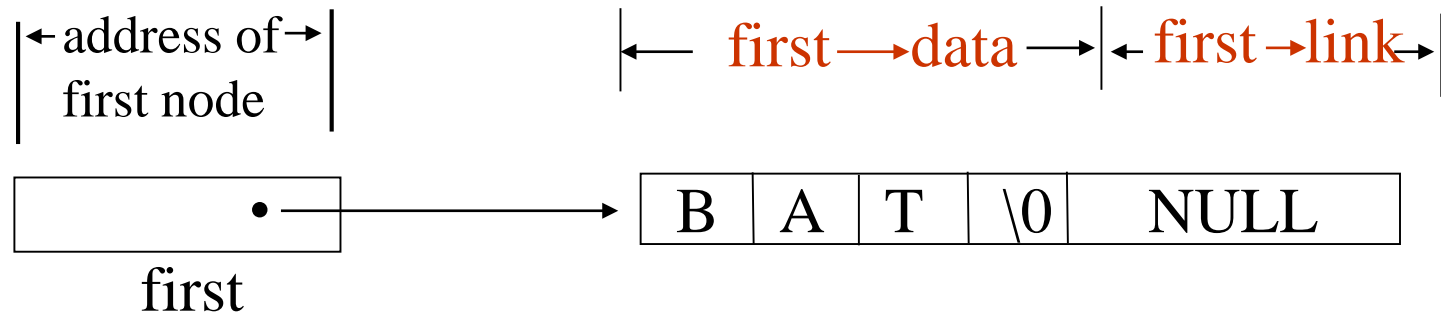
```
#define IS_EMPTY(first) (!(first))
```

## Allocation

```
first=(list_pointer) malloc (sizeof(list_node));
```

```
strcpy(first -> data, "BAT");  
first -> link = NULL;
```

$first \rightarrow data \Leftrightarrow (*first).data$



**\*Figure 4.5: Referencing the fields of a node**

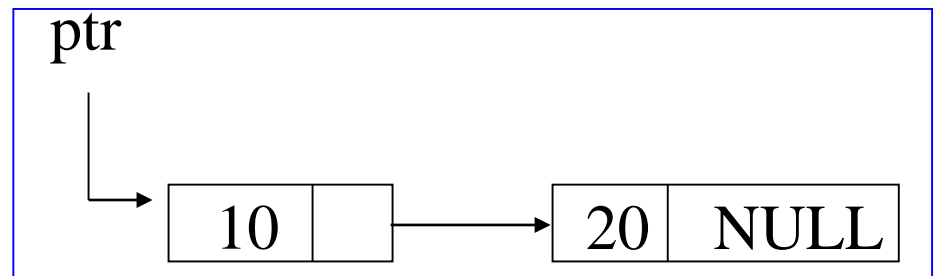
# Create a linked list pointer

```
typedef struct list_node *list_pointer;  
typedef struct list_node {  
    int data;  
    list_pointer link;  
};  
list_pointer ptr =NULL
```

ptr → NULL

# Create a two-node list

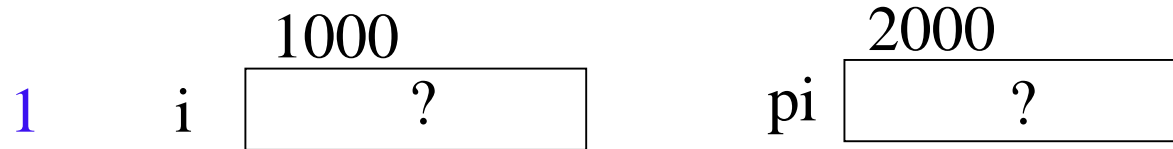
```
list_pointer create2( )
{
/* create a linked list with two nodes */
list_pointer first, second;
first = (list_pointer) malloc(sizeof(list_node));
second = ( list_pointer) malloc(sizeof(list_node));
second -> link = NULL;
second -> data = 20;
first -> data = 10;
first ->link = second;
return first;
}
```



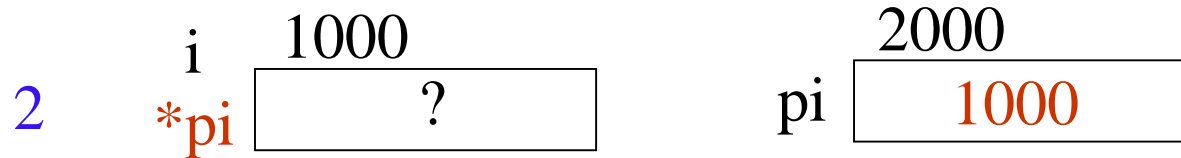
\*Program 4.1: Create a two-node list

# Pointer Review (1)

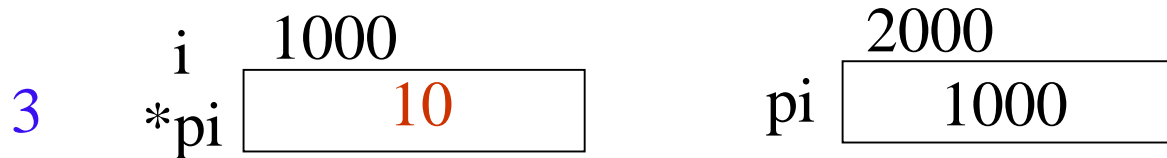
```
int i, *pi;
```



```
pi = &i;
```



```
i = 10 or *pi = 10
```



# Pointer Review (2)

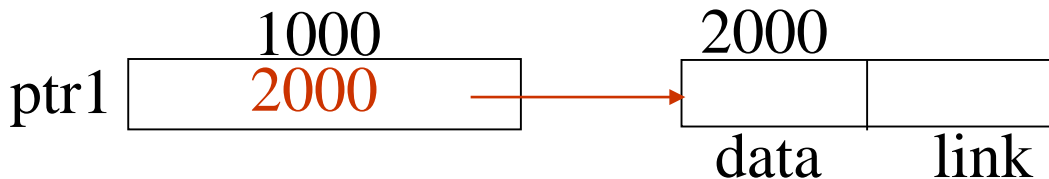
```
typedef struct list_node *list_pointer;  
typedef struct list_node {  
    int data;  
    list_pointer link;  
}
```

```
list_pointer ptr1 = NULL;
```



ptr1->data ⇔ (\*ptr1).data

```
ptr2 = malloc(sizeof(list_node));  
ptr1 = &ptr2;
```

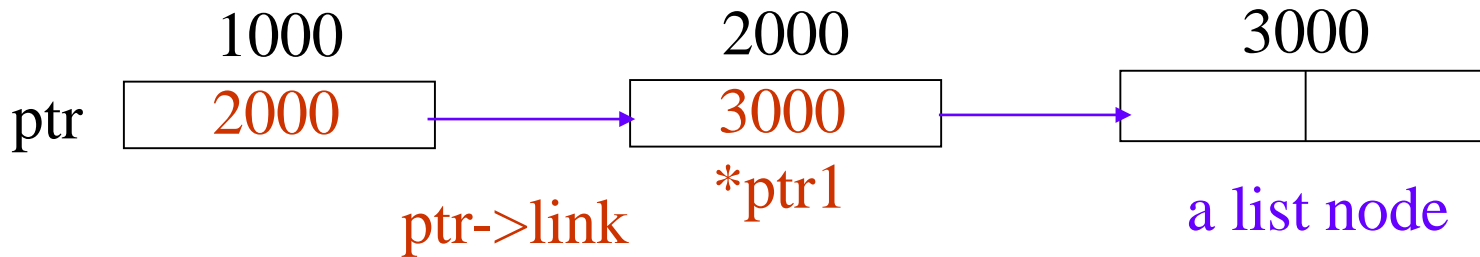


ptr2

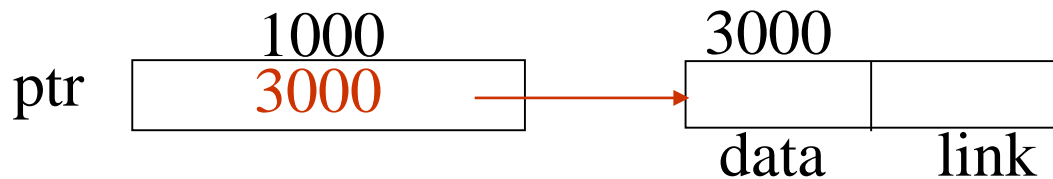
# Pointer Review (3)

```
void delete(list_pointer *ptr, list_pointer trail, list_pinter node)
```

ptr: a pointer point to a pointer point to a list node



ptr = & node; (a pointer point to a list node)







# List Insertion

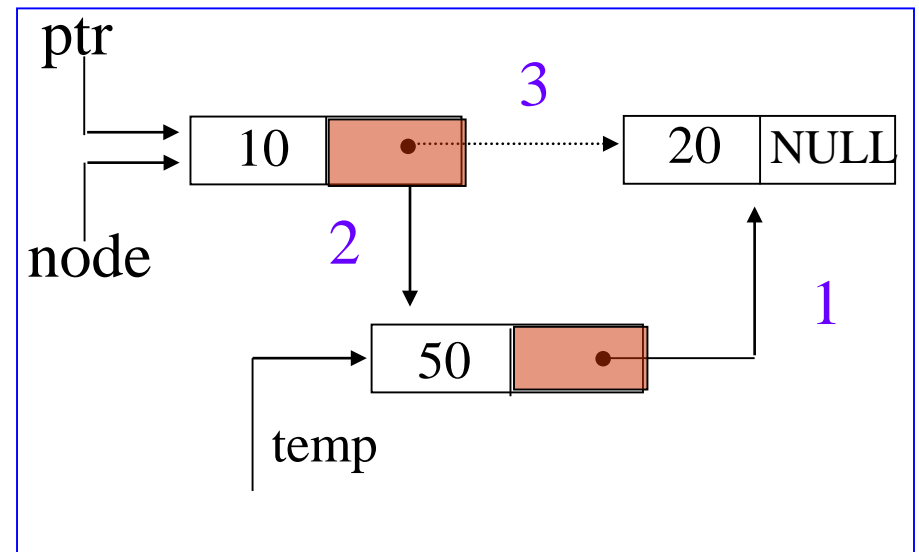
## Insert a node after a specific node

```
void insert(list_pointer *ptr, list_pointer x)
{
    /* insert a new node with data = 50 into the list ptr after node */
    list_pointer temp;
    temp = (list_pointer) malloc(sizeof(list_node));
    if (IS_FULL(temp)){
        fprintf(stderr, "The memory is full\n");
        exit (1);
    }
}
```

```

temp->data = 50;
if (*ptr) { //noempty list
    temp->link = node ->link;
    node->link = temp;
}
else { //empty list
    temp->link = NULL;
    *ptr =temp;
}
}

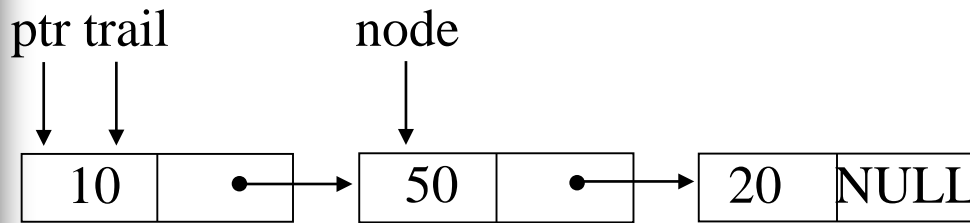
```



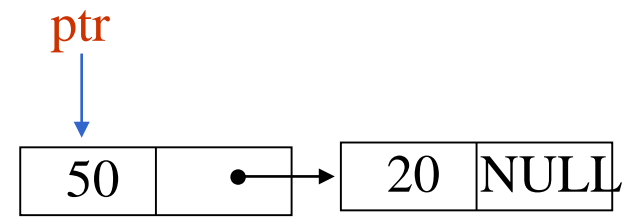
**\*Program 4.2: Simple insert into front of list**

# List Deletion

1: Delete the first node.

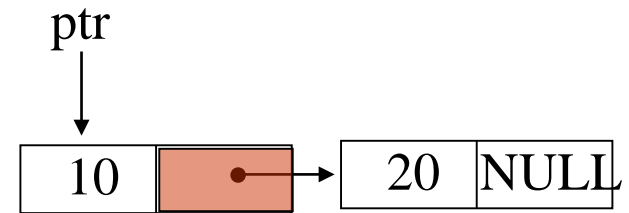
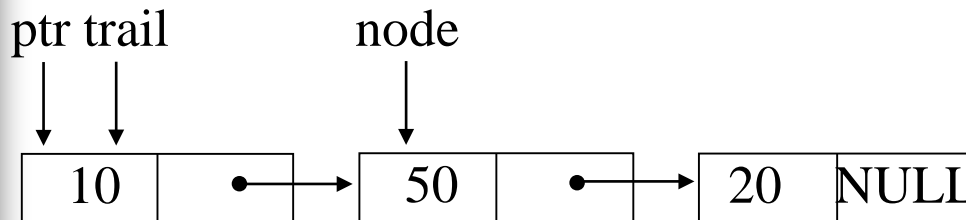


(a) before deletion



(b) after deletion

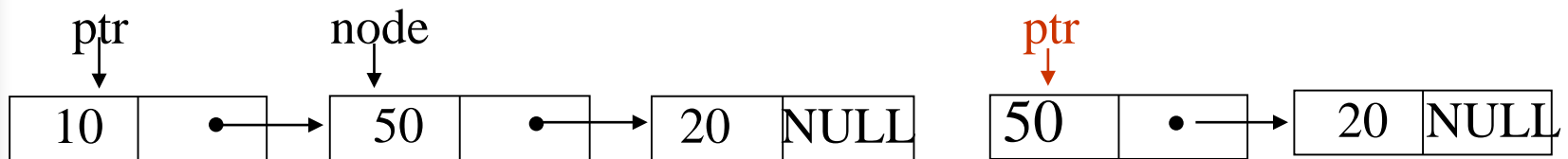
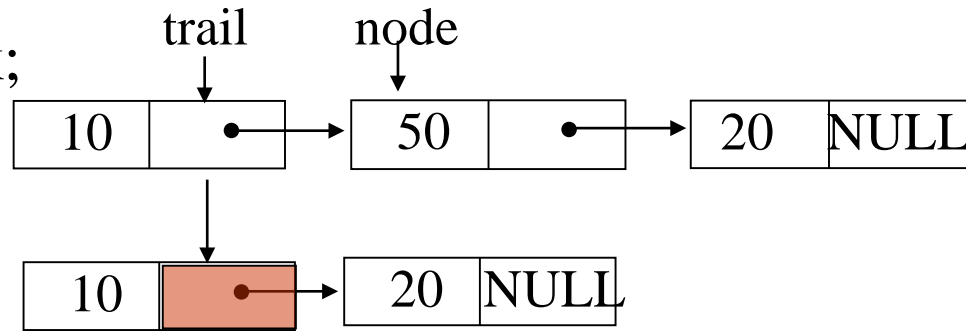
2: Delete node other than the first node.



```

void delete(list_pointer *ptr, list_pointer trail,
           list_pointer node)
{
/* delete node from the list, trail is the preceding node
ptr is the head of the list */
if (trail)
    trail->link = node->link;
else
    *ptr = ptr ->link; //head
free(node);
}

```



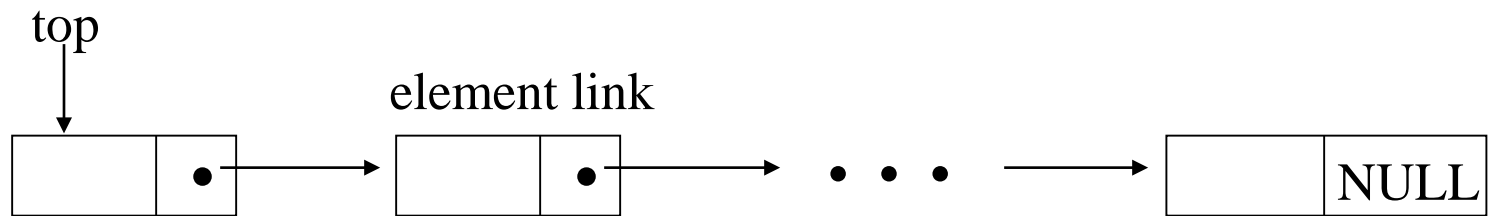


# Print out a list (traverse a list)

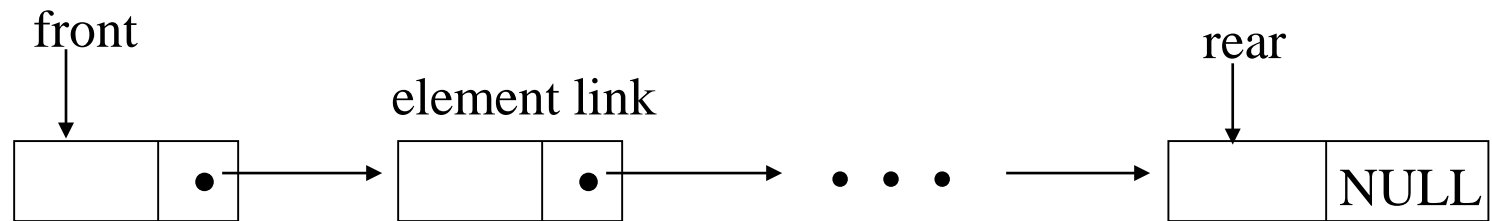
```
void print_list(list_pointer ptr)
{
    printf("The list contains: ");
    for ( ; ptr; ptr = ptr->link)
        printf("%4d", ptr->data);
    printf("\n");
}
```

**\*Program 4.4: Printing a list**

# Linked Stacks and Queues



(a) Linked Stack



(b) Linked queue

\*Figure 4.11: Linked Stack and queue

# Represent n stacks

```
#define MAX_STACKS 10 /* maximum number of stacks */
typedef struct {
    int key;
    /* other fields */
} element;
typedef struct stack *stack_pointer;

typedef struct stack {
    element item;
    stack_pointer link;
};
stack_pointer top[MAX_STACKS];
```

# Represent n queues

```
#define MAX_QUEUES 10 /* maximum number of queues */
typedef struct queue *queue_pointer;

typedef struct queue {
    element item;
    queue_pointer link;
};
queue_pointer front[MAX_QUEUE], rear[MAX_QUEUES];
```



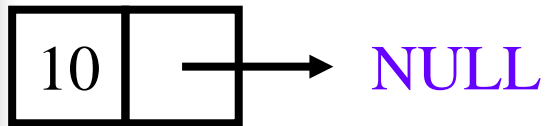


# Implementation stack by linked lists

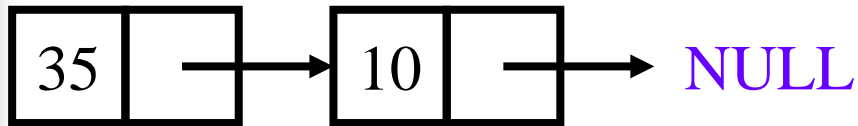
- 用list實作stack的插入和刪除功能
- 實作步驟
  1. 建立node的結構(struct)
  2. 主要function
    - 插入: push()
    - 刪除: pop()

# Implementation stack by linked lists

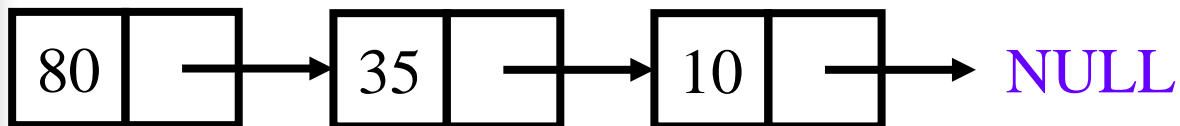
- push 10



- push 35

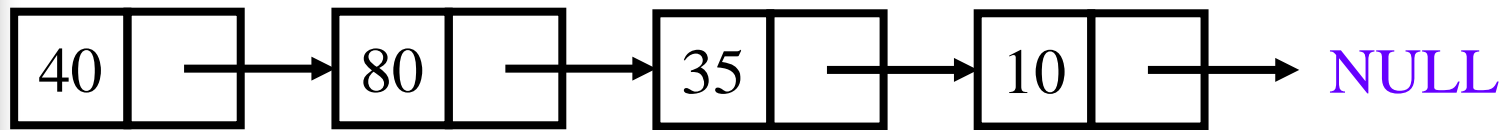


- push 80

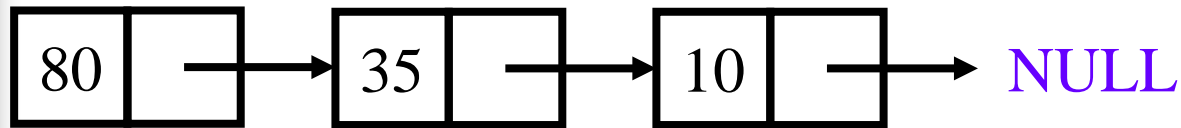


# Implementation stack by linked lists

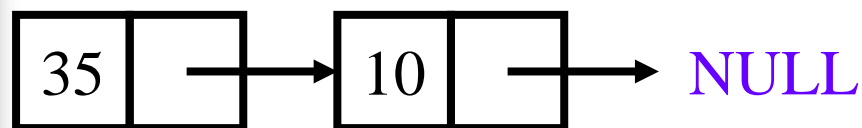
■ push 40



■ pop



■ pop



# push in the linked stack

```
void push(stack_pointer *top, element item)
{
    /* add an element to the top of the stack */
    stack_pointer temp =
        (stack_pointer) malloc (sizeof (stack));
    if (IS_FULL(temp)) {
        fprintf(stderr, " The memory is full\n");
        exit(1);
    }
    temp->item = item;
    temp->link = *top;
    *top= temp;
}
```

**\*Program 4.5: Add to a linked stack**

# pop from the linked stack

```
element pop(stack_pointer *top) {
/* delete an element from the stack */
    stack_pointer temp = *top;
    element item;
    if (IS_EMPTY(temp)) {
        fprintf(stderr, "The stack is empty\n");
        exit(1);
    }
    item = temp->item;
    *top = temp->link;
    free(temp);
    return item;
}
```

**\*Program 4.6:** Delete from a linked stack

# Implementation stack by linked lists

- 建立node的結構(struct)

```
struct stackNode{  
    int data;  
    struct stackNode *nextPtr;  
};  
typedef struct stackNode StackNode; // synonym for struct stackNode  
typedef StackNode *StackNodePtr;    // synonym for StackNode*
```

# Implementation stack by linked lists

- 插入: push()

```
void push(StackNodePtr *topPtr, int info){
    StackNodePtr newPtr;
    newPtr = malloc(sizeof(StackNode));
    if(newPtr != NULL){
        newPtr->data = info;
        newPtr->nextPtr = *topPtr;
        *topPtr = newPtr;
    }
    else{
        printf("%d not inserted. No memory available.\n", info);
    }
}
```

# Implementation stack by linked lists

- 插入: pop()

```
int pop(StackNodePtr *topPtr){
    StackNodePtr tempPtr;
    int popValue;
    tempPtr = *topPtr;
    popValue = (*topPtr)->data;
    *topPtr = (*topPtr)->nextPtr;
    free(tempPtr);
    return popValue;
}
```



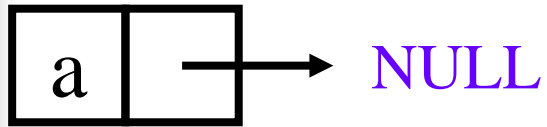


# Implementation queue by linked lists

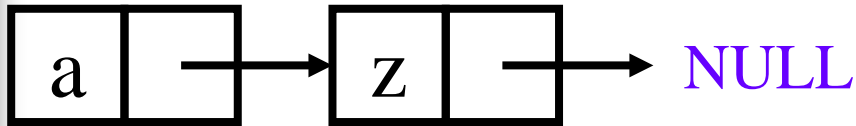
- 用list實作Queue的插入和刪除功能
- 實作步驟
  1. 建立node的結構(struct)
  2. 主要function
    - 插入: enqueue()
    - 刪除: dequeue()

# Implementation queue by linked lists

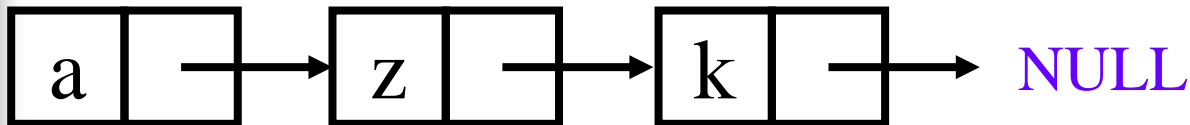
- enqueue a



- enqueue z

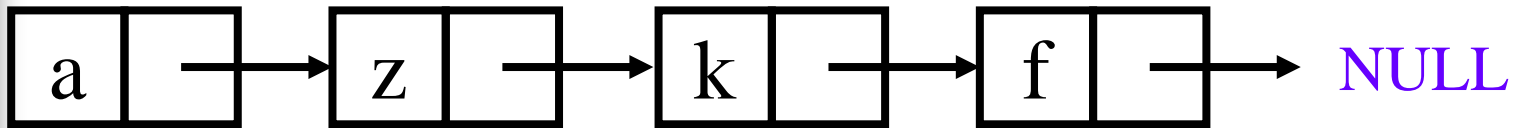


- enqueue k

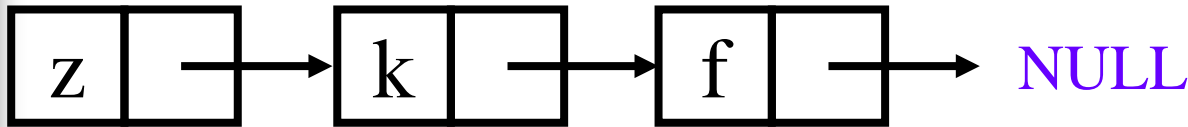


# Implementation queue by linked lists

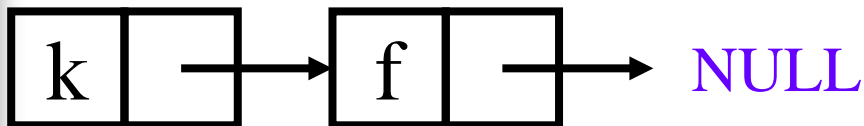
- enqueue f



- dequeue



- dequeue



# enqueue in the linked queue

```
void addq(queue_pointer *front, queue_pointer *rear, element  
item)
```

```
{ /* add an element to the rear of the queue */  
  queue_pointer temp =  
      (queue_pointer) malloc(sizeof (queue));  
  if (IS_FULL(temp)) {  
      fprintf(stderr, “ The memory is full\n”);  
      exit(1);  
  }  
  temp->item = item;  
  temp->link = NULL;  
  if (*front)  
      rear -> link = temp;  
  else *front = temp;  
  *rear = temp; }
```

# dequeue from the linked queue

```
element deleteq(queue_pointer *front) {
/* delete an element from the queue */
    queue_pointer temp = *front;
    element item;
    if (IS_EMPTY(*front)) {
        fprintf(stderr, "The queue is empty\n");
        exit(1);
    }
    item = temp->item;
    *front = temp->link;
    free(temp);
    return item;
}
```

# Implementation queue by linked lists

- 建立node的結構(struct)

```
struct queueNode{
    char data;
    struct queueNode *nextPtr;
};
typedef struct queueNode QueueNode; // synonym for struct queueNode
typedef QueueNode *QueueNodePtr;    // synonym for QueueNode*
```

# Implementation queue by linked lists

## ■ 插入: enqueue()

```
void enqueue(QueueNodePtr *headPtr, QueueNodePtr *tailPtr, char value){
    QueueNodePtr newPtr;
    newPtr = malloc(sizeof(QueueNode));
    if(newPtr != NULL){
        newPtr->data = value;
        newPtr->nextPtr = NULL;
        if(isEmpty(*headPtr)){
            *headPtr = newPtr;           // the queue is empty
        }else{
            (*tailPtr)->nextPtr = newPtr; // the queue isn't empty
        }
        *tailPtr = newPtr;
    }
    else{
        printf("%c not inserted. No memory available.\n", value);
    }
}
```

# Implementation queue by linked lists

- 删除: dequeue()

```
char dequeue(QueueNodePtr *headPtr, QueueNodePtr *tailPtr){
    char value;
    QueueNodePtr tempPtr;
    value = (*headPtr)->data;
    tempPtr = *headPtr;
    *headPtr = (*headPtr)->nextPtr;
    if(*headPtr == NULL){
        *tailPtr = NULL;
    }
    free(tempPtr);
    return value;
}
```



# Polynomials

$$A(x) = a_{m-1}x^{e_{m-1}} + a_{m-2}x^{e_{m-2}} + \dots + a_0x^{e_0}$$

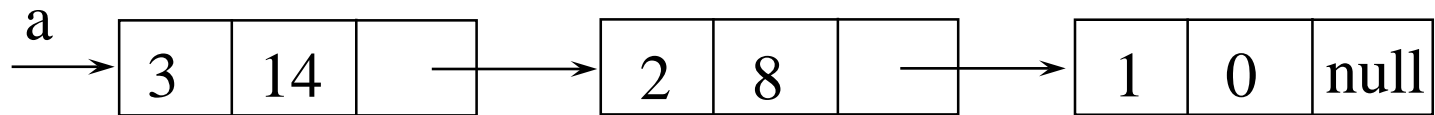
## Representation

```
typedef struct poly_node *poly_pointer;
typedef struct poly_node {
    int coef;
    int expon;
    poly_pointer link;
};
poly_pointer a, b, c;
```

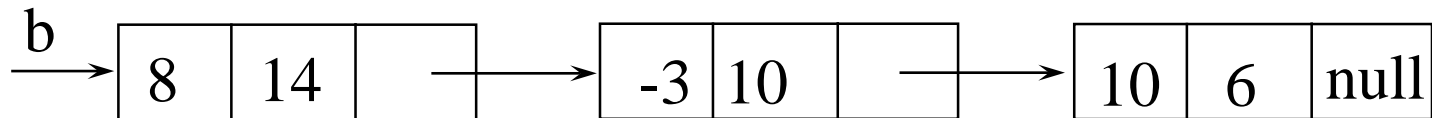
coef	expon	link
------	-------	------

# Examples

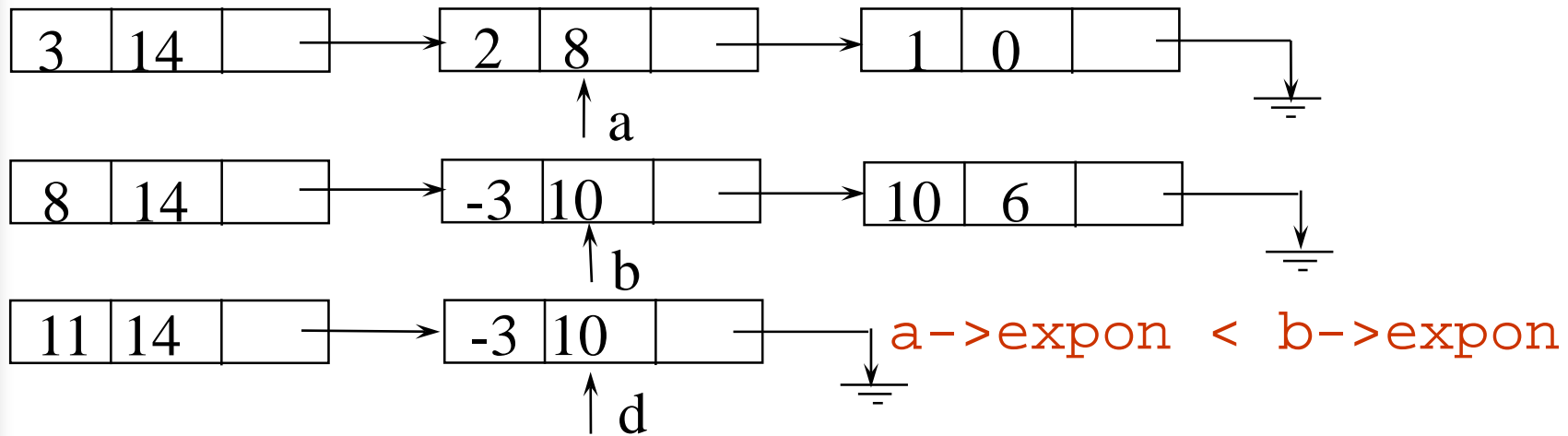
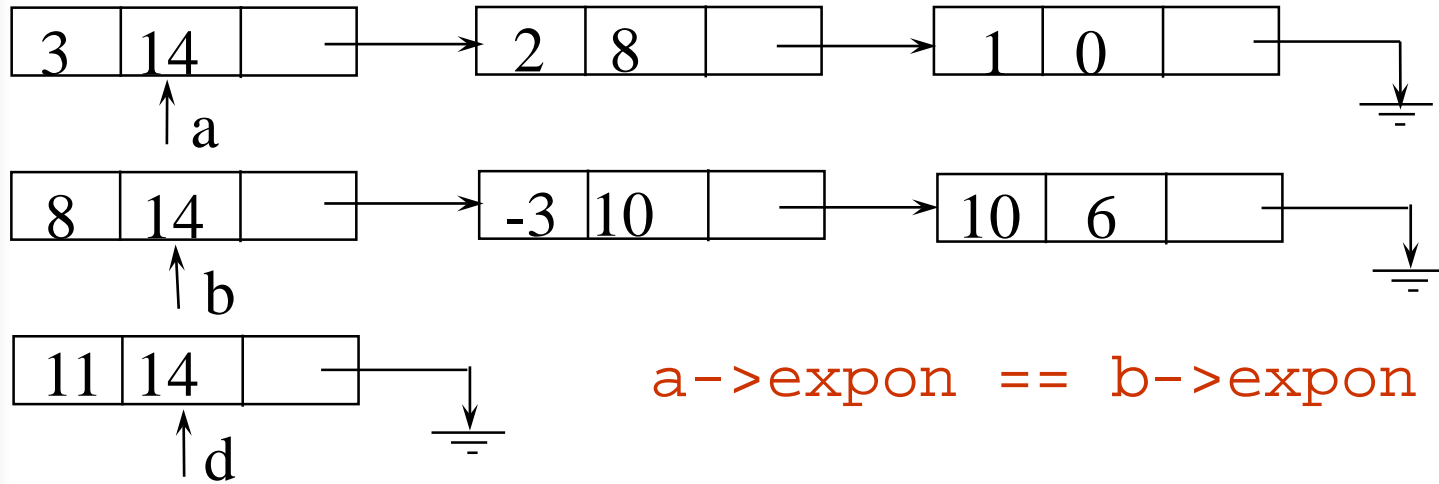
$$a = 3x^{14} + 2x^8 + 1$$



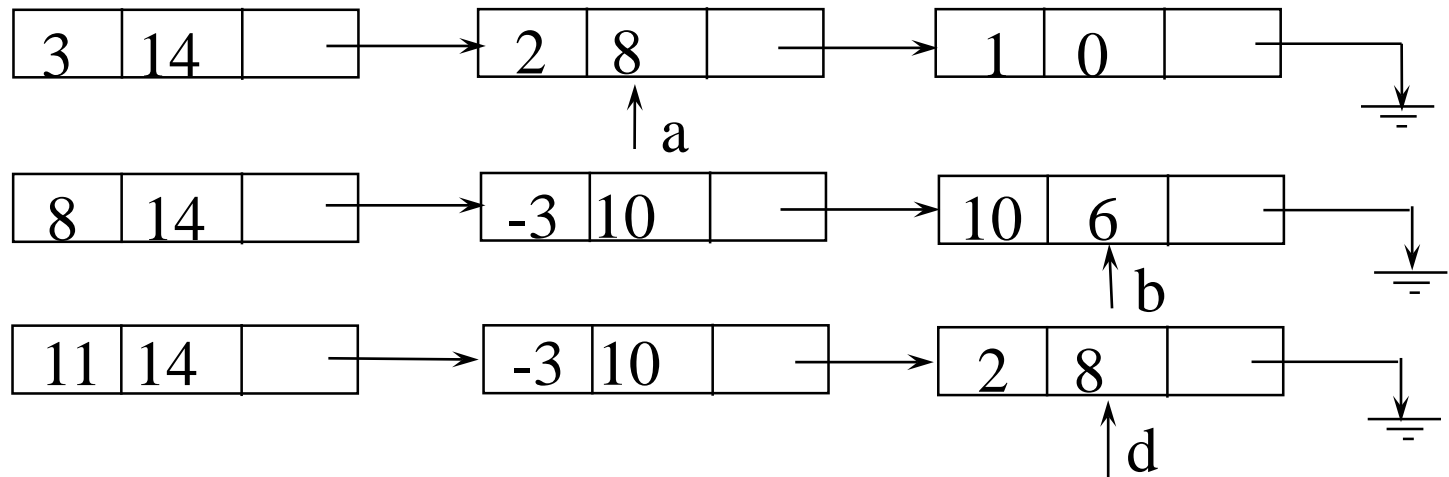
$$b = 8x^{14} - 3x^{10} + 10x^6$$



# Adding Polynomials



# Adding Polynomials (*Continued*)



a->expon > b->expon

# Algorithm for Adding Polynomials

```
poly_pointer padd(poly_pointer a, poly_pointer b)
{
    poly_pointer c, rear, temp;
    int sum;
    rear = (poly_pointer)malloc(sizeof(poly_node));
    if (IS_FULL(rear)) {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    front = rear;
    while (a && b) {
        switch (COMPARE(a->expon, b->expon)) {
```

```

    case -1: /* a->expon < b->expon */
        attach(b->coef, b->expon, &rear);
        b = b->link;
        break;
    case 0: /* a->expon == b->expon */
        sum = a->coef + b->coef;
        if (sum) attach(sum, a->expon, &rear);
        a = a->link;    b = b->link;
        break;
    case 1: /* a->expon > b->expon */
        attach(a->coef, a->expon, &rear);
        a = a->link;
    }
}
for (; a; a = a->link)
    attach(a->coef, a->expon, &rear);
for (; b; b = b->link)
    attach(b->coef, b->expon, &rear);
rear->link = NULL;
temp = front; front = front->link; free(temp);
return front;
}

```

Delete extra initial node.

# Attach a Term

```
void attach(float coefficient, int exponent,
            poly_pointer *ptr)
{
    /* create a new node attaching to the node pointed to
       by ptr. ptr is updated to point to this new node. */
    poly_pointer temp;
    temp = (poly_pointer) malloc(sizeof(poly_node));
    if (IS_FULL(temp)) {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    temp->coef = coefficient;
    temp->expon = exponent;
    ptr ->link = temp;
    *ptr = temp;
}
```

# Algorithm for adding polynomials

- 建立多項式node的結構(struct)

```
struct polyNode{
    int coef;
    int expon;
    struct polyNode *link;
};
typedef struct polyNode PolyNode; // synonym for struct polyNode
typedef PolyNode *PolyNodePtr; // synonym for PolyNode*
PolyNodePtr a, b;
```



# Algorithm for adding polynomials

## ■ padd()

```
PolyNodePtr padd(PolyNodePtr a, PolyNodePtr b){
    PolyNodePtr c, rear, temp;
    int sum;
    MALLOC(rear, sizeof(*rear));
    c = rear;
    while(a && b){
        switch(COMPARE(a->expon, b->expon)){
            case -1: // a->expon < b->expon
                attach(b->coef, b->expon, &rear);
                b = b->link;
                break;
            case 0: // a->expon = b->expon
                sum = a->coef + b->coef;
                if (sum)
                    attach(sum, a->expon, &rear);
                a = a->link;
                b = b->link;
            case 1: // a->expon > b->expon
                attach(a->coef, a->expon, &rear);
                a = a->link;
        }
    }
}
```

# Algorithm for adding polynomials

- padd()

```
// copy rest of list a and then list b
for(; a; a = a->link)
    attach(a->coef, a->expon, &rear);
for(; b; b = b->link)
    attach(b->coef, b->expon, &rear);
rear->link = NULL;
// delete extra initial node
temp = c;
c = c->link;
free(temp);
return c;
}
```

# Algorithm for adding polynomials

- attach()

```
void attach(float coefficient, int exponent, PolyNodePtr *ptr){
    PolyNodePtr temp;
    MALLOC(temp, sizeof(*temp));
    temp->coef = coefficient;
    temp->expon = exponent;
    (*ptr)->link = temp;
    *ptr = temp;
}
```

# Analysis

(1) coefficient additions

$0 \leq \text{number of coefficient additions} \leq \min(m, n)$

where  $m$  ( $n$ ) denotes the number of terms in  $A$  ( $B$ )

(2) exponent comparisons

extreme case

$$e_{m-1} > f_{m-1} > e_{m-2} > f_{m-2} > \dots > e_0 > f_0$$

$m+n-1$  comparisons

(3) creation of new nodes

extreme case

$m + n$  new nodes  $O(m+n)$

summary

# A Suite for Polynomials

$$e(x) = a(x) * b(x) + d(x)$$

```
poly_pointer a, b, d, e;
```

```
...
```

```
a = read_poly();
```

```
b = read_poly();
```

```
d = read_poly();
```

```
temp = pmult(a, b);
```

```
e = padd(temp, d);
```

```
print_poly(e);
```

```
read_poly()  
print_poly()  
  
padd()  
psub()  
pmult()
```

temp is used to hold a partial result.  
By returning the nodes of temp, we  
may use it to hold other polynomials

# Erase Polynomials

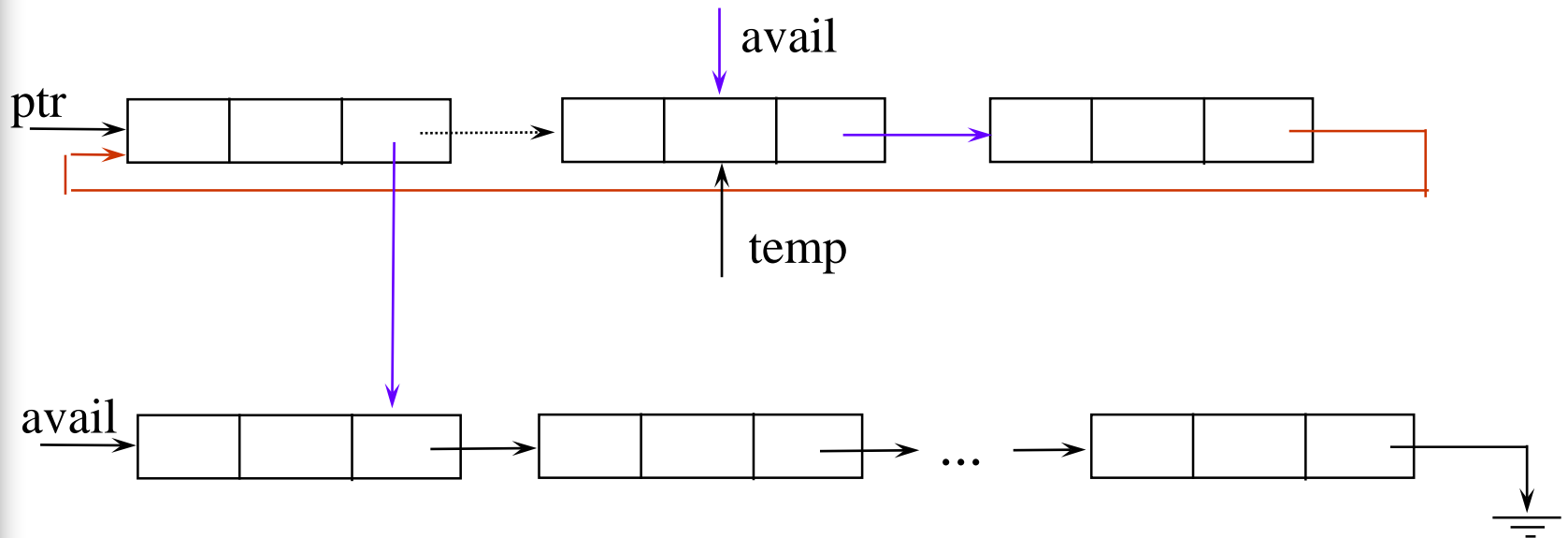
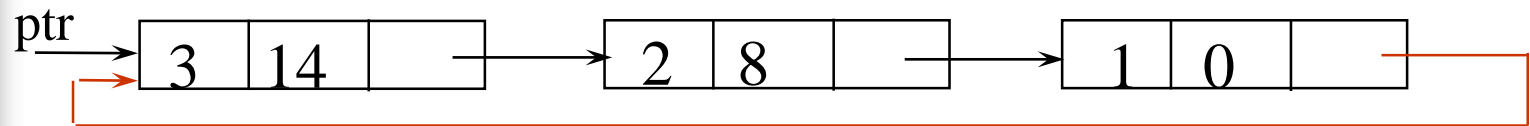
```
void earse(poly_pointer *ptr)
{
/* erase the polynomial pointed to by ptr */
    poly_pointer temp;

    while (*ptr) {
        temp = *ptr;
        *ptr = ptr->link;
        free(temp);
    }
}
```

$O(n)$

# Circularly Linked Lists

circular list vs. chain



# Maintain an Available List

```
poly_pointer getnode(void)
{
    poly_pointer node;
    if (avail) {
        node = avail;
        avail = avail->link;
    }
    else {
        node = (poly_pointer)malloc(sizeof(poly_node));
        if (IS_FULL(node)) {
            printf(stderr, "The memory is full\n");
            exit(1);
        }
    }
    return node;
}
```



# Maintain an Available List *(Continued)*

```
void retNode(poly_pointer ptr)
{
    ptr->link = avail;
    avail = ptr;
}
```

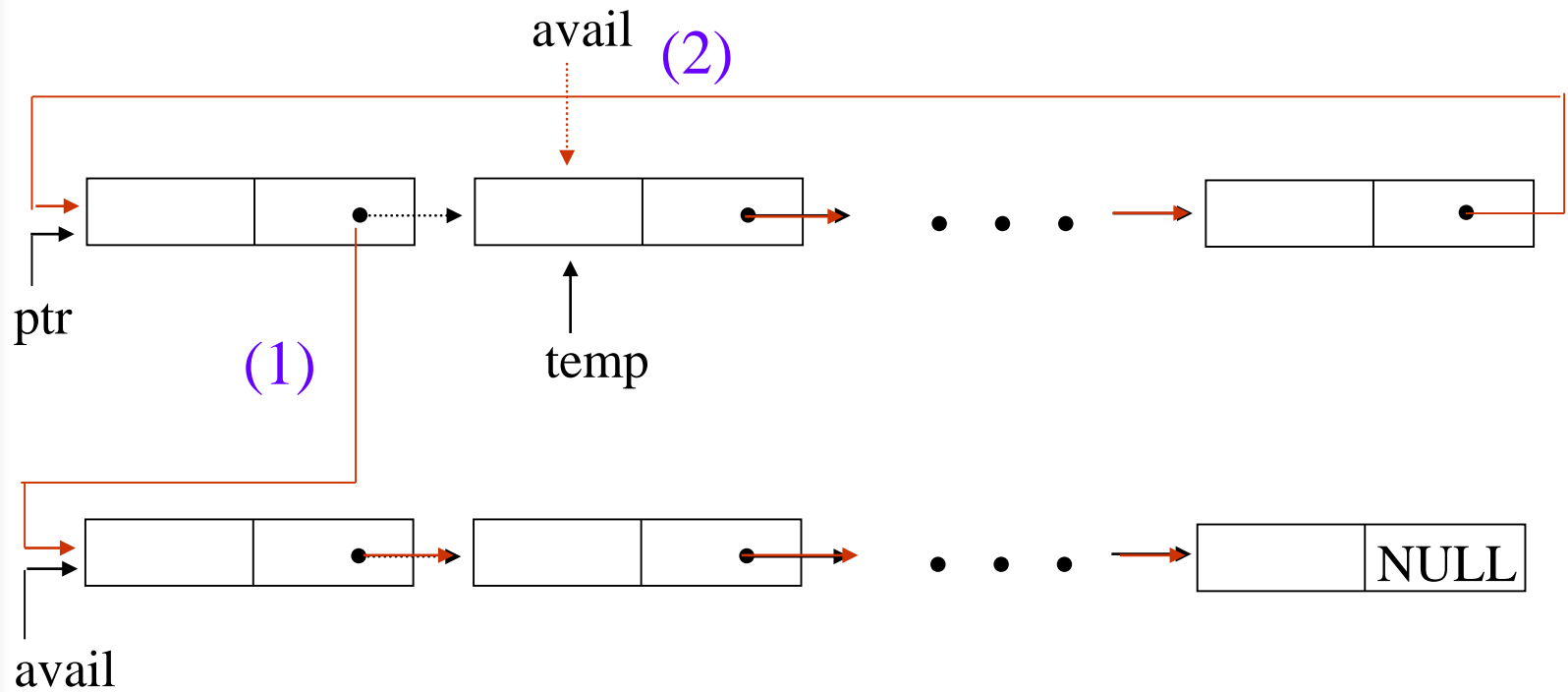
```
void cerase(poly_pointer *ptr)
{
    poly_pointer temp;
    if (*ptr) {
        temp = ptr->link;
        ptr->link = avail;
        avail = temp;
        *ptr = NULL;
    }
}
```

← (1)  
← (2)

Erase a circular list (see next page)

Independent of # of nodes in a list **O(1) constant time**

# Circular List Representing of Polynomials

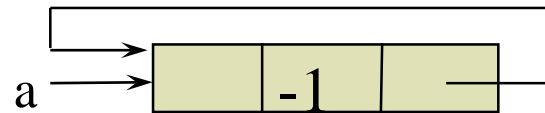


Returning a circular list to the avail list

# Head Node

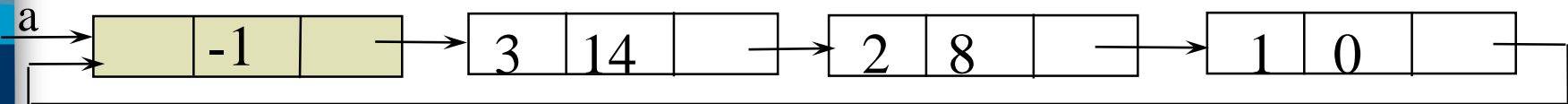
Represent **polynomial** as **circular list**.

(1) zero



Zero polynomial

(2) others



$$a = 3x^{14} + 2x^8 + 1$$

# Another Padd

```
poly_pointer cpadd(poly_pointer a, poly_pointer b)
{
    poly_pointer startA, c, lastC;
    int sum, done = FALSE;
    starta = a;
    a = a->link;
    b = b->link;
    c = getnode();
    c->expon = -1;      lastC = c;
    /* get a header node for a and b*/
    do {
        switch (COMPARE(a->expon, b->expon)) {
            case -1: attach(b->coef, b->expon, &lastC);
                    b = b->link;
                    break;
        }
    }
}
```

Set expon field of head node to -1.

## Another Padd (*Continued*)

```
case 0: if (startA == a) done = TRUE;
        else {
            sum = a->coef + b->coef;
            if (sum) attach(sum, a->expon, &lastC);
            a = a->link;    b = b->link;
        }
        break;
case 1: attach(a->coef, a->expon, &lastC);
        a = a->link;
    }
} while (!done);
lastC->link = c;
return c;
```

➔ **Link last node to first**



# Additional List Operations

```
typedef struct list_node *list_pointer;  
typedef struct list_node {  
    char data;  
    list_pointer link;  
};
```

Invert single linked lists


Concatenate two linked lists

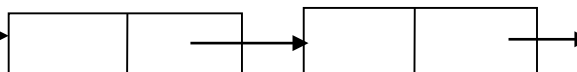
# Invert Single Linked Lists

Use two extra pointers: middle and trail

```
list_pointer invert(list_pointer lead)
{
    list_pointer middle, trail;
    middle = NULL;
    while (lead) {
        trail = middle; /* NULL */
        middle = lead;
        lead = lead->link;
        middle->link = trail;
    }
    return middle;
}
```

0: null

1: lead →  ...

≥2: lead → 

```
middle = NULL
```

```
While(lead){
```

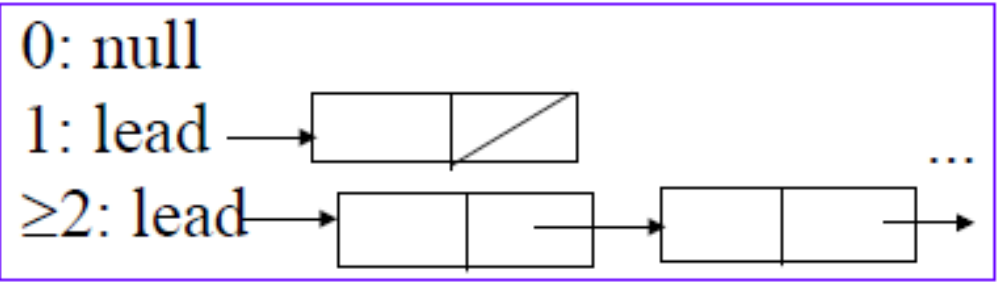
```
    trail = middle
```

```
    middle = lead
```

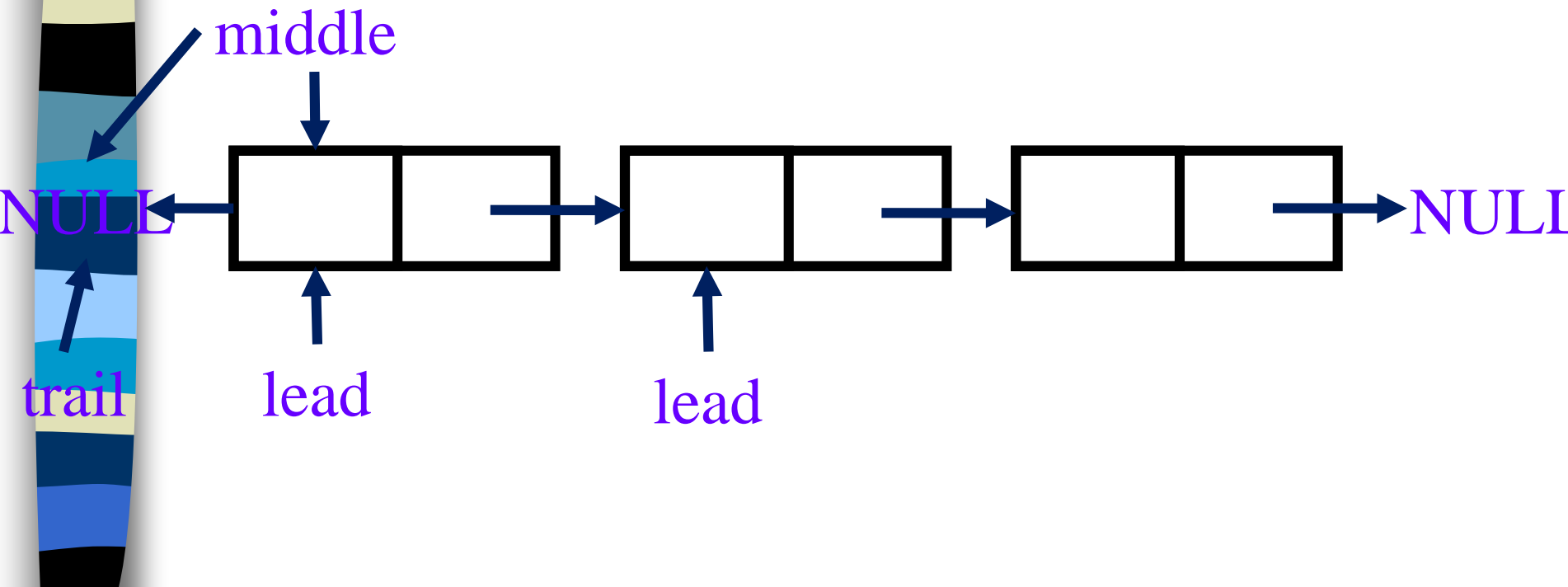
```
    lead = lead -> link
```

```
    middle -> link = trail
```

```
}
```



Round 1





middle = NULL

While(lead){

trail = middle

middle = lead

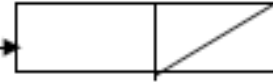
lead = lead -> link

middle -> link = trail

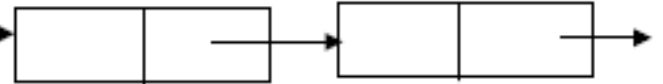
}

0: null

1: lead →



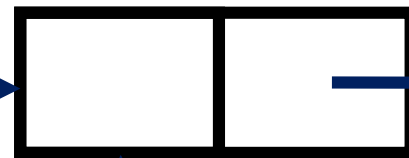
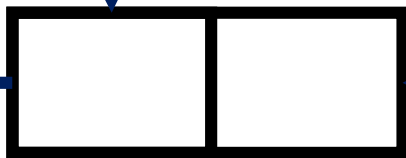
≥2: lead →



Round 2

middle

middle



NULL

trail

lead

lead

NULL

trail

middle = NULL

```
While(lead){
```

```
  trail = middle
```

```
  middle = lead
```

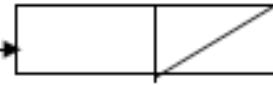
```
  lead = lead -> link
```

```
  middle -> link = trail
```

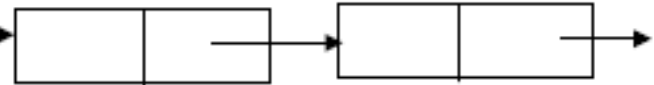
```
}
```

0: null

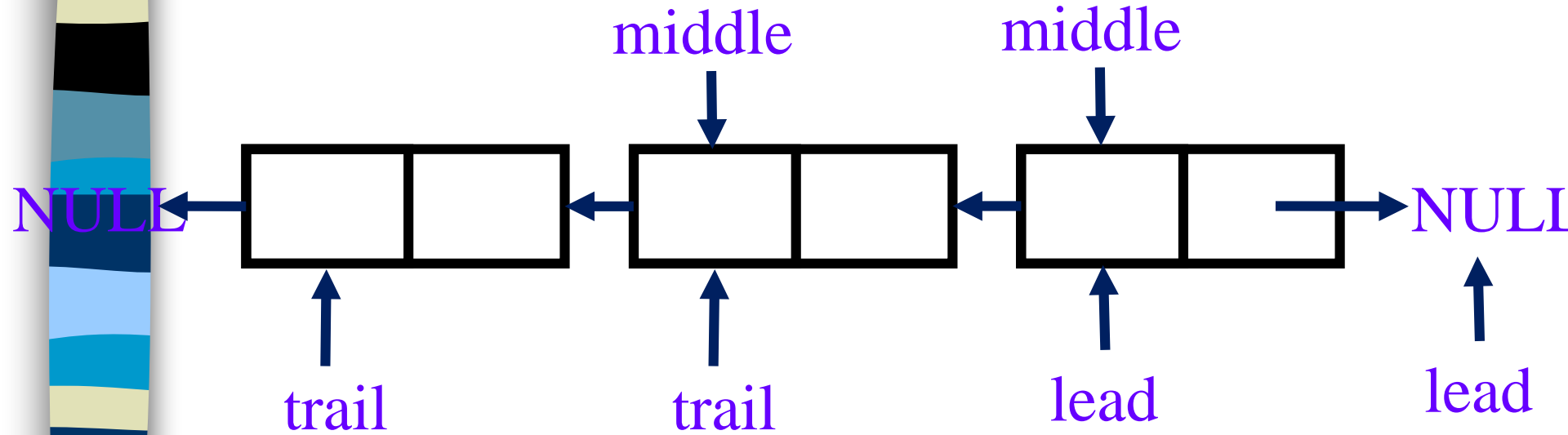
1: lead →



≥2: lead →



### Round 3



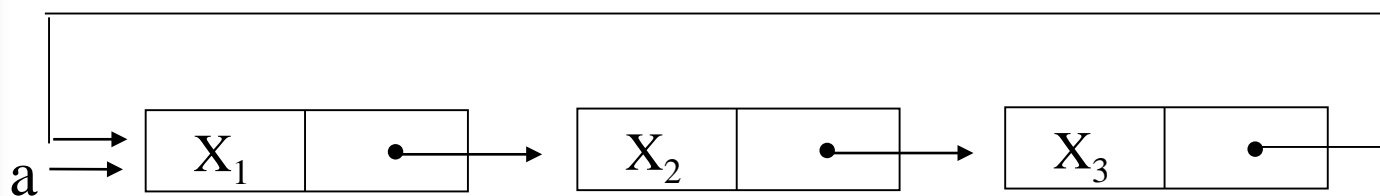
# Concatenate Two Lists

```
list_pointer concatenate(list_pointer
                        ptr1, list_pointer ptr2)
{
    list_pointer temp;
    if (IS_EMPTY(ptr1)) return ptr2;
    else {
        if (!IS_EMPTY(ptr2)) {
            for (temp=ptr1;temp->link;temp=temp->link);
/*find end of first list*/
            temp->link = ptr2;
        }
        return ptr1;
    }
}
```

$O(m)$  where  $m$  is # of elements in the first list

# Operations For Circularly Linked List

What happens when we insert a node to the front of a circular linked list?

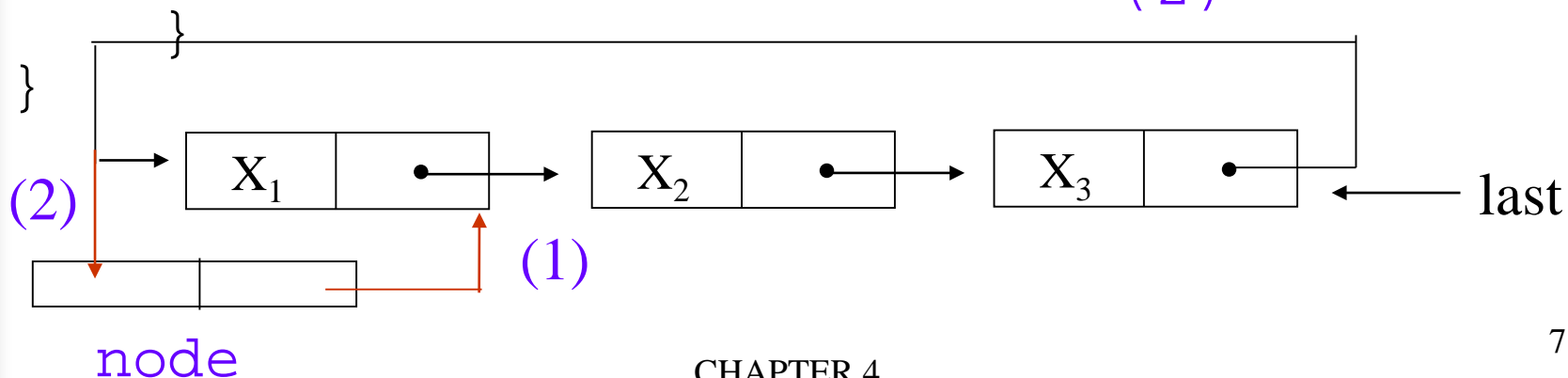


Problem: move down the whole list.

\*Figure 4.16: Example circular list

# Operations for Circular Linked Lists

```
void insertFront(list_pointer *last, list_pointer
node)
{
    if (!(*last)) {
        /* list is empty, change last to point to new
entry*/
        *last= node;
        node->link = node;
    }
    else {
        node->link = (*last)->link; (1)
        (*last)->link = node; (2)
    }
}
```



# Length of Linked List

```
int length(list_pointer last)
{
    list_pointer temp;
    int count = 0;
    if (last) {
        temp = last;
        do {
            count++;
            temp = temp->link;
        } while (temp!=last);
    }
    return count;
}
```



# Equivalence Relations

A relation over a set,  $S$ , is said to be an *equivalence relation* over  $S$  iff it is **symmetric**, **reflexive**, and **transitive** over  $S$ .

reflexive,  $x=x$

symmetric, if  $x=y$ , then  $y=x$

transitive, if  $x=y$  and  $y=z$ , then  $x=z$



# Examples

$0 \equiv 4, 3 \equiv 1, 6 \equiv 10, 8 \equiv 9, 7 \equiv 4,$   
 $6 \equiv 8, 3 \equiv 5, 2 \equiv 11, 11 \equiv 0$

three equivalent classes

$\{0,2,4,7,11\}; \{1,3,5\}; \{6,8,9,10\}$



# A Rough Algorithm to Find Equivalence Classes

```
void equivalenec()  
{  
    initialize;  
    while (there are more pairs) {  
        read the next pair <i,j>;  
        process this pair;  
    }  
    initialize the output;  
    do {  
        output a new equivalence class;  
    } while (not done);  
}
```

Phase 1

Phase 2

What kinds of data structures are adopted?

# First Refinement

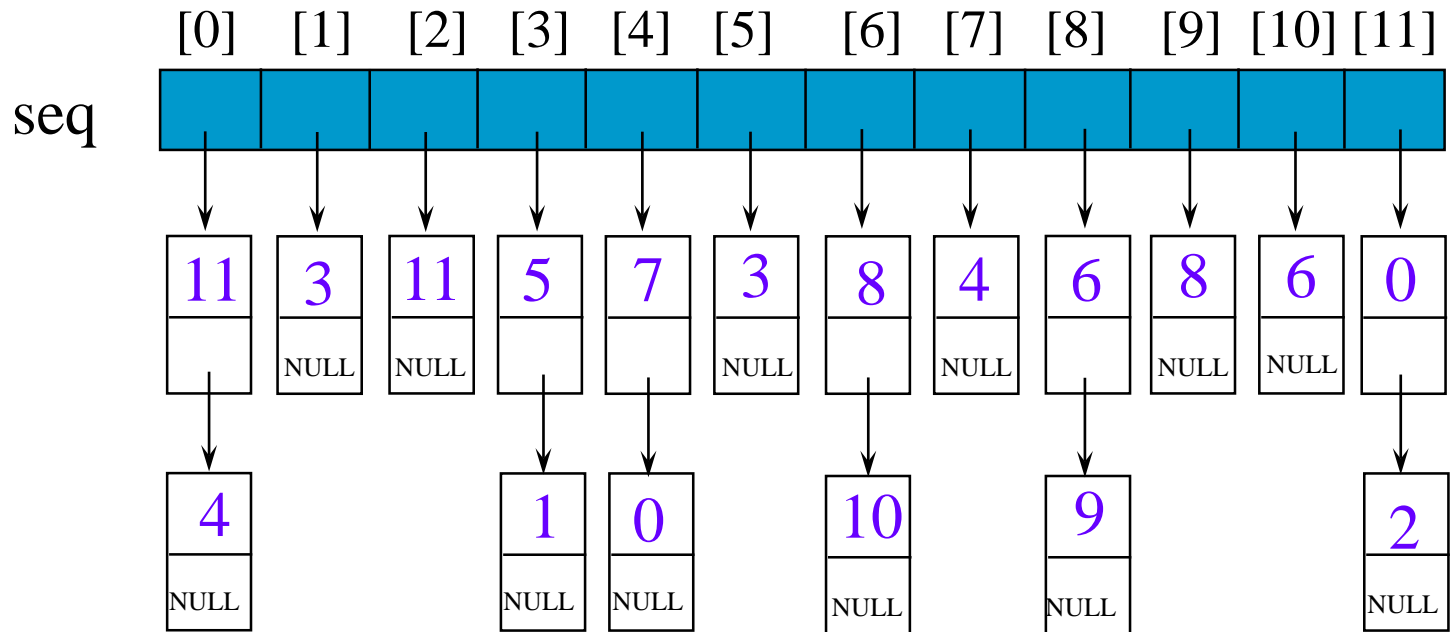
```
#include <stdio.h>
#include <alloc.h>
#define MAX_SIZE 24
#define IS_FULL(ptr) (!(ptr))
#define FALSE 0
#define TRUE 1
void equivalence()
{
    initialize seq to NULL and out to TRUE
    while (there are more pairs) {
        read the next pair <i,j>
        put j on the seq[i] list;
        put i on the seq[j] list;
    }
    for (i=0; i<n; i++)
        if (out[i]) {
            out[i]= FALSE;
            output this equivalence class;
        }
}
```

direct equivalence

Compute indirect equivalence  
using transitivity

# Lists After Pairs are input

0 ≡ 4  
3 ≡ 1  
6 ≡ 10  
8 ≡ 9  
7 ≡ 4  
6 ≡ 8  
3 ≡ 5  
2 ≡ 11  
11 ≡ 0

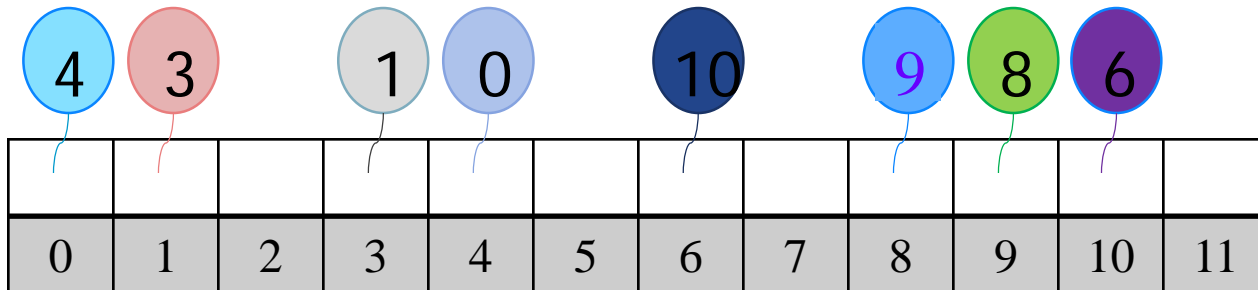


```
typedef struct node *node_pointer ;  
typedef struct node {  
    int data;  
    node_pointer link;  
};
```

# Program to find equivalence classes

First, we input many pair of numbers and

(0,4) (3,1) (6,10) (8,9)



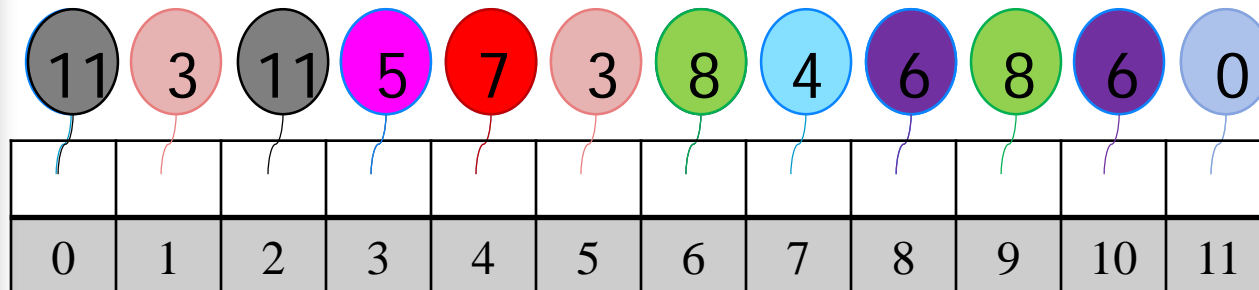
# Program to find equivalence classes

First, we input many pair of numbers and

$(0,4)(3,1)(6,10)(8,9)$

And add the number from the bottom if the space isn't empty

$(7,4)(6,8)(3,5)(2,11)(11,0)$



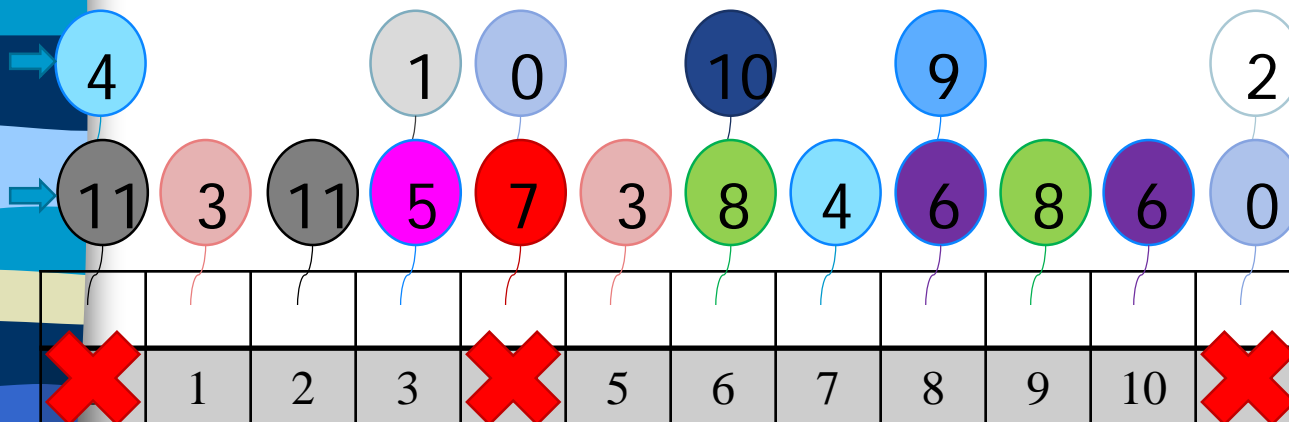
# Program to find equivalence classes

From 0 to 11 if the number is unused, print "New class", print and mark the number become used.

Print:  
New class: 0 11 4

And from its link to find another number, if the linked number also link another number, push the linked number to stack.

Then, from the top of stack to find the same class number.



out[i] stack

# Program to find equivalence classes

From 0 to 11 if the number is unused, print "New class", print and mark the number become used.

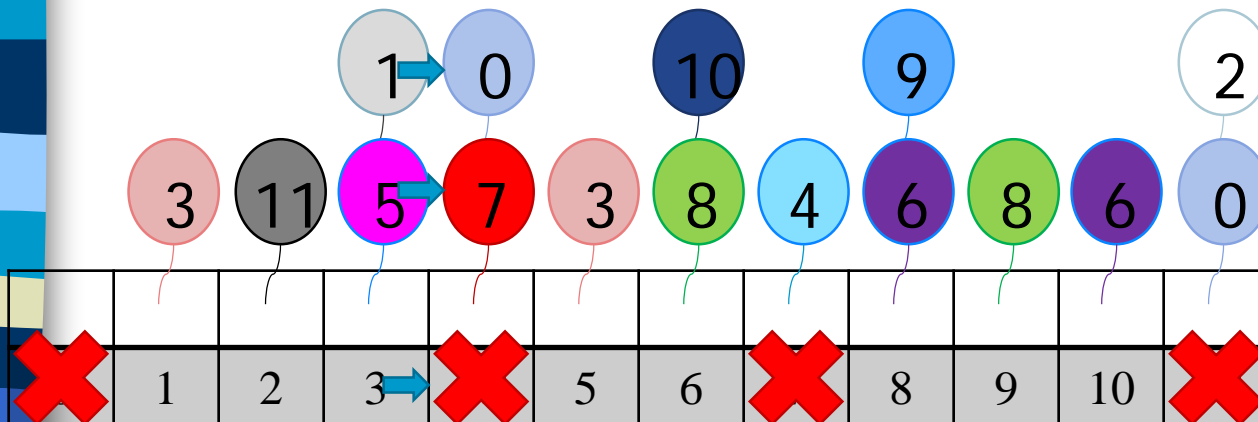
Print:

New class: 0 11 4 7

And from its link to find another number, if the linked number also link another number, push the linked number to stack.

Then, from the top of stack to find the same class number.

If the number is already used, skip it.



stack

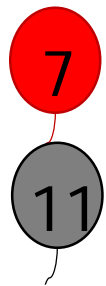
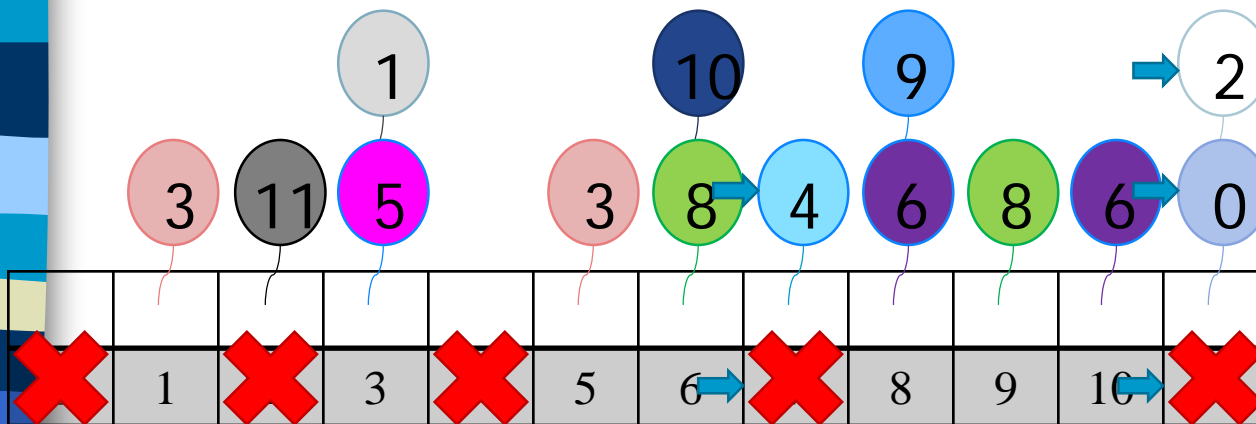
# Program to find equivalence classes

From 0 to 11 if the number is unused, print "New class", print and mark the number become used.

Print:

New class: 0 11 4 7 2

And from its link to find another number, if the linked number also link another number, push the linked number to stack. Then, from the top of stack to find the same class number. If the number is already used, skip it.



stack



# Program to find equivalence classes

From 0 to 11 if the number is unused, print "New class", print and mark the number become used.

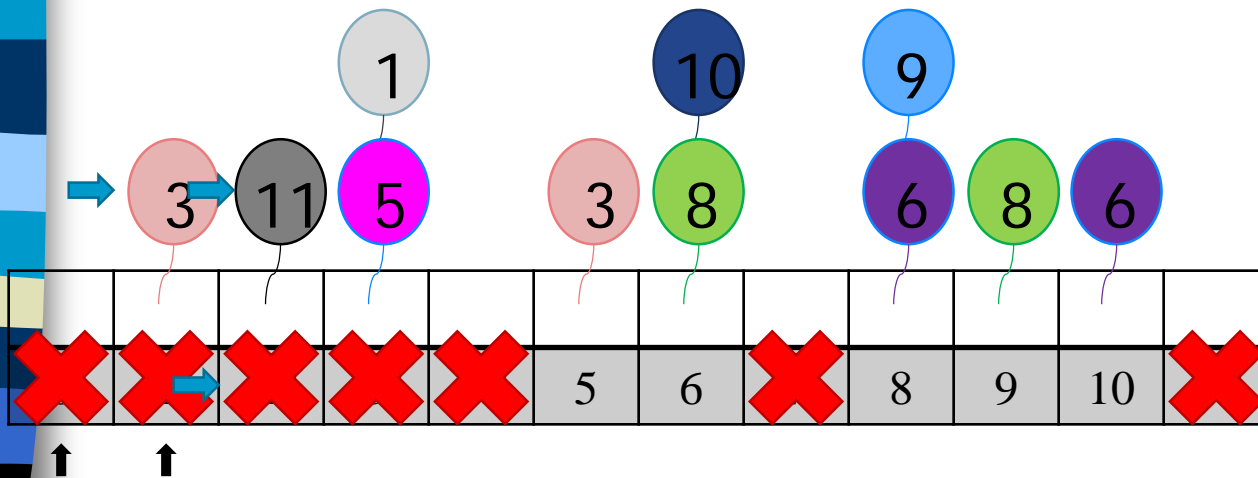
Print:

New class: 0 11 4 7 2

New class: 1 3

And from its link to find another number, if the linked number also link another number, push the linked number to stack.

Then, from the top of stack to find the same class number. If the number is already used, skip it.



# Program to find equivalence classes

From 0 to 11 if the number is unused, print "New class", print and mark the number become used.

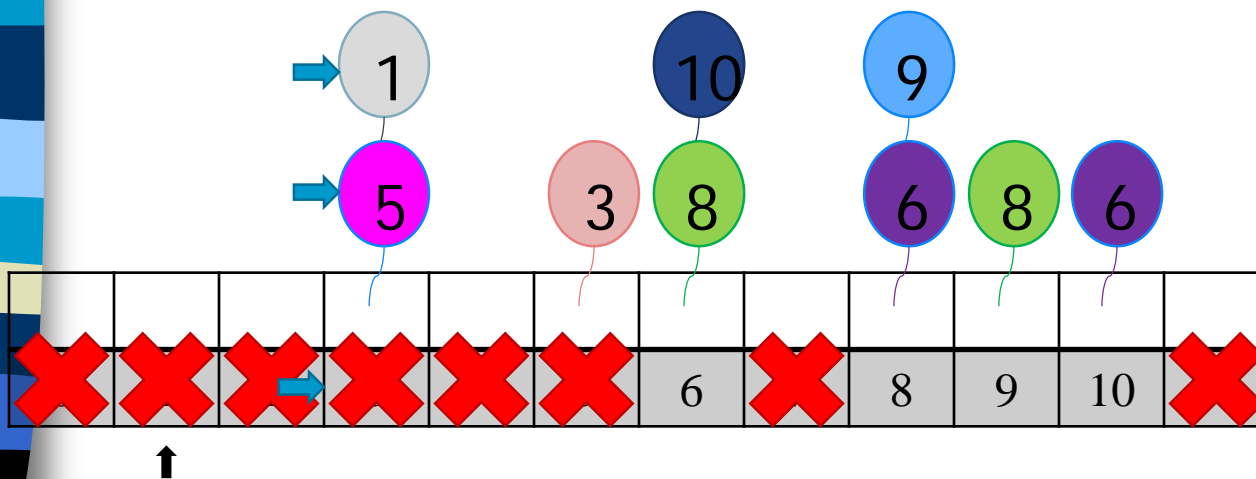
Print:

New class: 0 11 4 7 2

New class: 1 3 5

And from its link to find another number, if the linked number also link another number, push the linked number to stack.

Then, from the top of stack to find the same class number. If the number is already used, skip it.



# Program to find equivalence classes

From 0 to 11 if the number is unused, print "New class", print and mark the number become used.

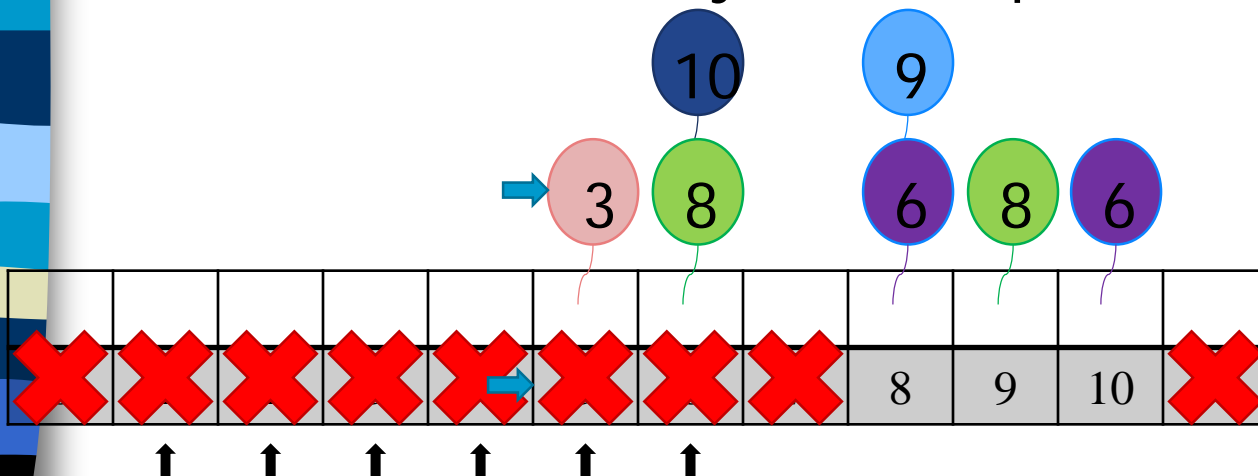
Print:

New class: 0 1 1 4 7 2

New class: 1 3 5

New class: 6

And from its link to find another number,  
if the linked number also link another number,  
push the linked number to stack.  
Then, from the top of stack to find the same class number.  
If the number is already used, skip it.



stack

# Program to find equivalence classes

From 0 to 11 if the number is unused, print "New class", print and mark the number become used.

Print:

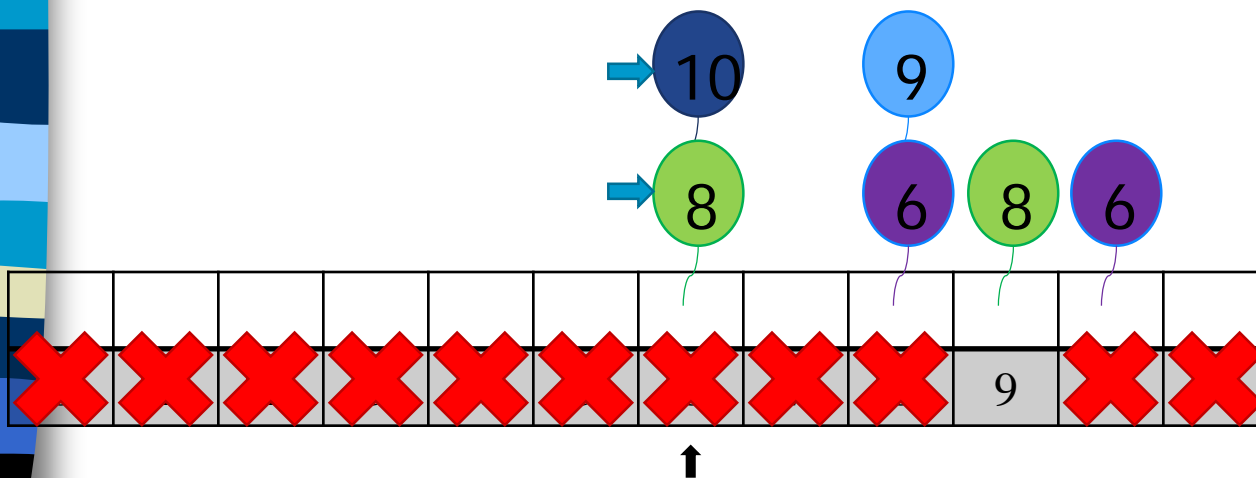
New class: 0 11 4 7 2

New class: 1 3 5

New class: 6 8 10

And from its link to find another number, if the linked number also link another number, push the linked number to stack.

Then, from the top of stack to find the same class number. If the number is already used, skip it.



stack

# Program to find equivalence classes

From 0 to 11 if the number is unused, print "New class", print and mark the number become used.

Print:

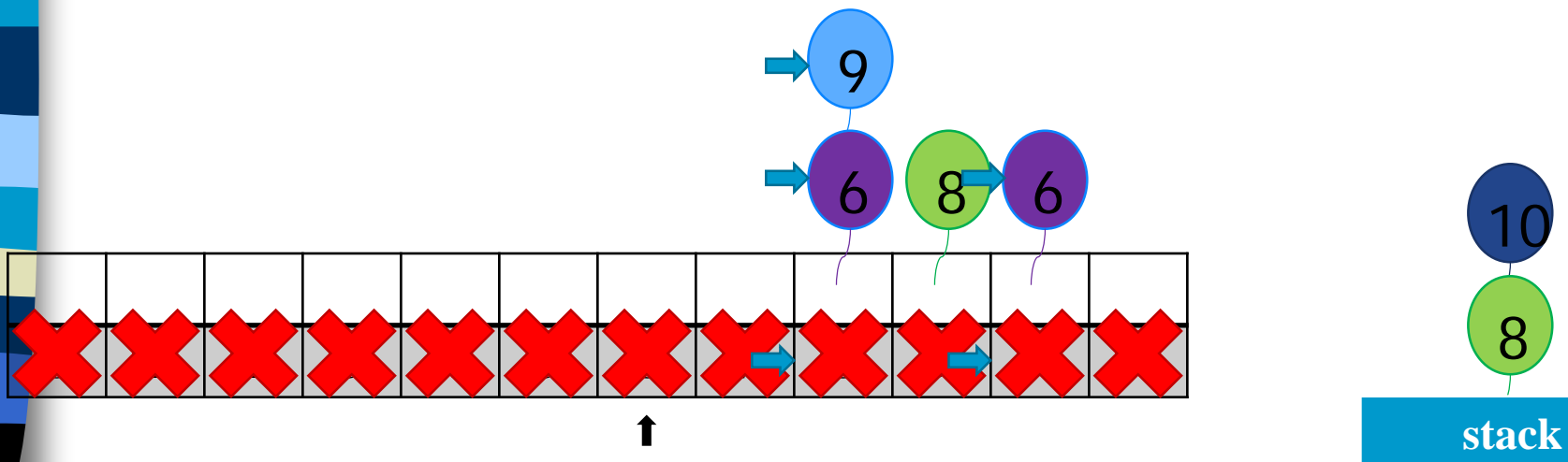
New class: 0 11 4 7 2

New class: 1 3 5

New class: 6 8 10 9

And from its link to find another number, if the linked number also link another number, push the linked number to stack.

Then, from the top of stack to find the same class number. If the number is already used, skip it.



# Program to find equivalence classes

From 0 to 11 if the number is unused, print "New class", print and mark the number become used.

Print:

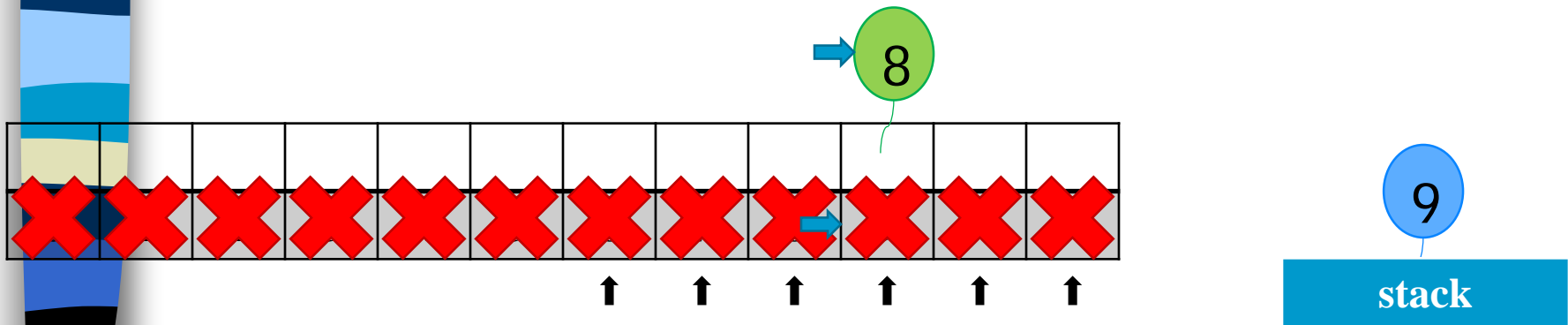
New class: 0 11 4 7 2

New class: 1 3 5

New class: 6 8 10 9

And from its link to find another number, if the linked number also link another number, push the linked number to stack.

Then, from the top of stack to find the same class number. If the number is already used, skip it.

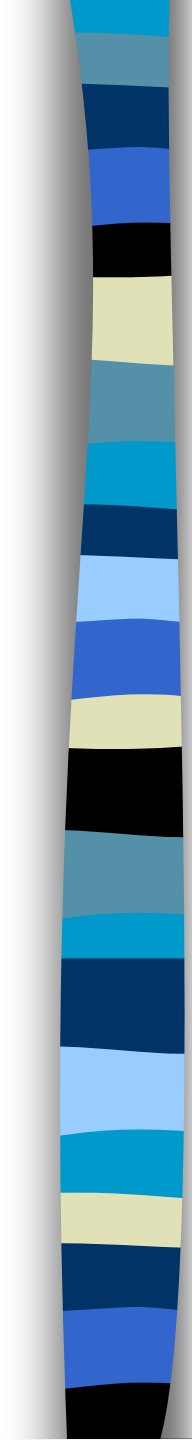




# Final Version for Finding Equivalence Classes

```
void main(void)
{
    short int out[MAX_SIZE];
    node_pointer seq[MAX_SIZE];
    node_pointer x, y, top;
    int i, j, n;
    printf("Enter the size (<= %d)", MAX_SIZE);
    scanf("%d", &n);
    for (i=0; i<n; i++) {
        out[i]= TRUE;      seq[i]= NULL;
    }
    printf("Enter a pair of numbers (-1 -1 to quit): ");
    scanf("%d%d", &i, &j);
}
```

Phase 1: input the equivalence pairs:



```
while (i>=0) {
    x = (node_pointer) malloc(sizeof(node));
    if (IS_FULL(x))
        fprintf(stderr, "memory is full\n");
        exit(1);
    } Insert x to the top of lists seq[i]
    x->data= j;  x->link= seq[i];  seq[i]= x;
    if (IS_FULL(x))
        fprintf(stderr, "memory is full\n");
        exit(1);
    } Insert x to the top of lists seq[j]
    x->data= i;  x->link= seq[j];  seq[j]= x;
    printf("Enter a pair of numbers (-1 -1 to \
        quit): ");
    scanf("%d%d", &i, &j);
}
```



## Phase 2: output the equivalence classes

```
for (i=0; i<n; i++)
{
    if (out[i]) {
        printf("\nNew class: %5d", i);
        out[i]= FALSE;
        x = seq[i];      top = NULL;
        for (j;
            {
                while (x)
                {
                    j = x->data;
                    if (out[j]) {           Move down
                        printf("%5d", j);
                        out[j] = FALSE;
                        y = x->link;  x->link = top;
                        top = x;  x = y;
                    }
                    else x = x->link; Next x
                }
                if (!top) break;
                x = seq[top->data];  top = top->link;
            }
        }
} // main
```

## 4.7 Sparse Matrices

$$\begin{bmatrix} 0 & 0 & 11 & 0 \\ 12 & 5 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & -15 \end{bmatrix}$$

inadequates of sequential schemes

- (1) # of nonzero terms will vary after some matrix computation
- (2) matrix just represents intermediate results

**New scheme**

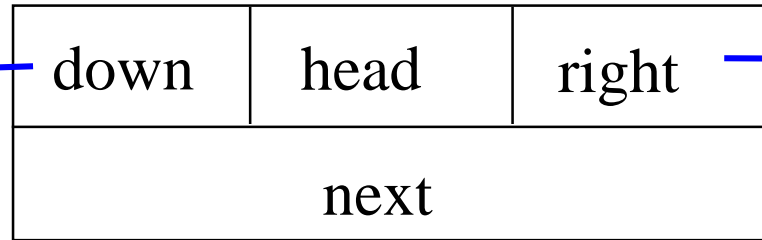
Each column (row): a circular linked list with a head node

# Revisit Sparse Matrices

# of head nodes = max{# of rows, # of columns}

head node

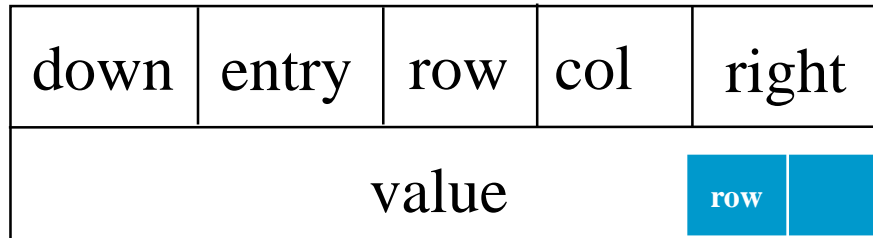
連同一行元素



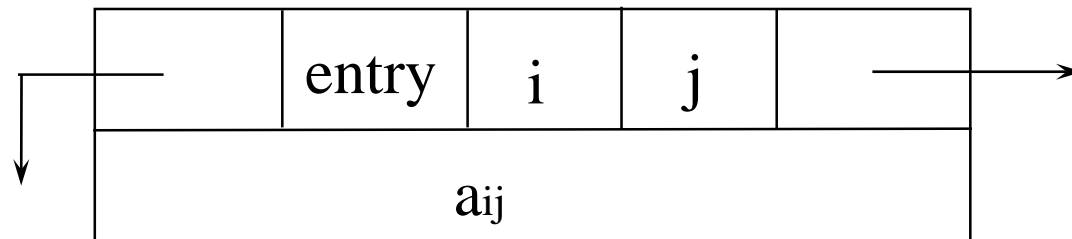
連同一列元素



entry node

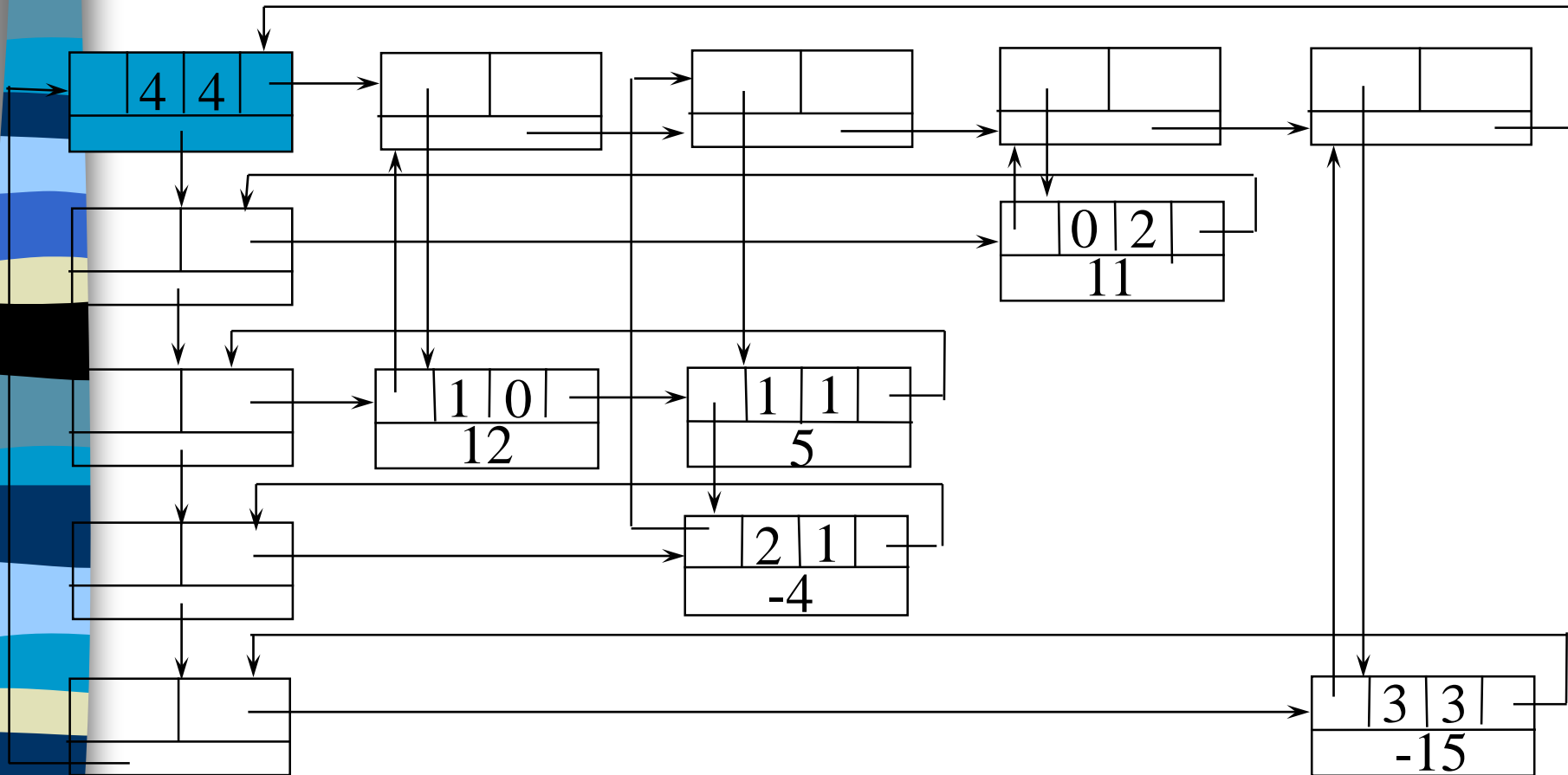


$a_{ij}$

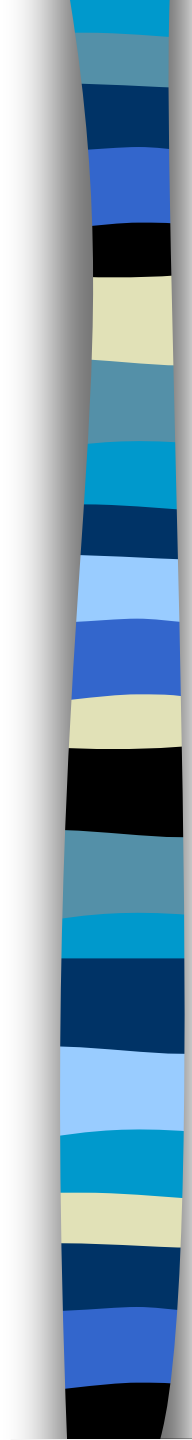


# Linked Representation for Matrix

Information

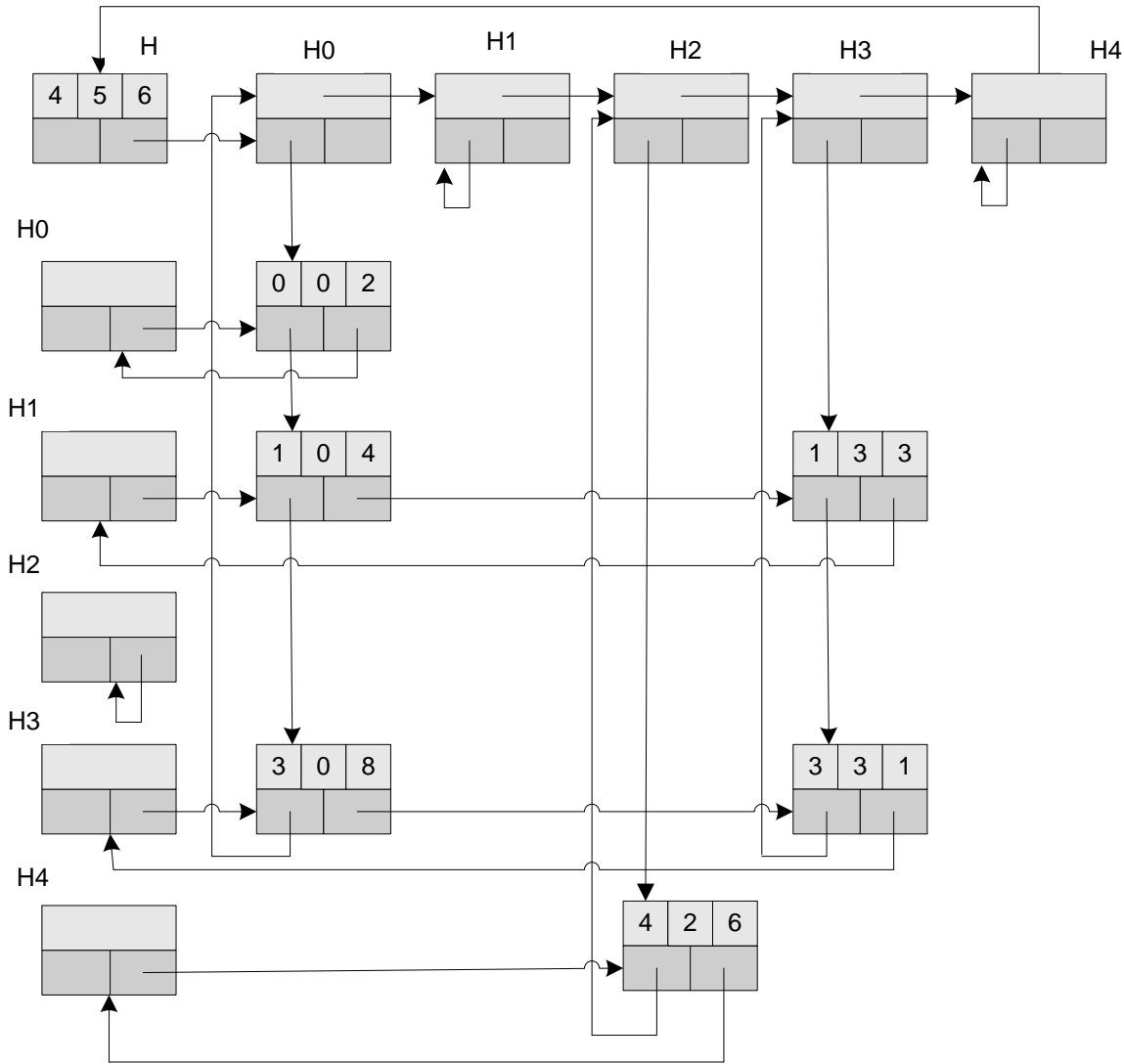


Circular linked list



```
#define MAX_SIZE 50 /* size of largest matrix */
typedef enum {head, entry} tagfield;
typedef struct matrixNode *matrixPointer;
typedef struct entryNode {
    int row;
    int col;
    int value;
};
typedef struct matrixNode {
    matrixPointer down;
    matrixPointer right;
    tagfield tag;    → head or entry
    union {
        matrixPointer next;
        entryNode entry;
    } u;
};
matrixPointer hdnode[MAX_SIZE];
```

# Information



$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 1 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$

# Read in a Matrix

```
matrix_pointer mread(void)
{
/* read in a matrix and set up its linked
list. An global array hdnnode is used */
int num_rows, num_cols, num_terms;
int num_heads, i;
int row, col, value, current_row;
matrixPointer temp, last, node;

printf("Enter the number of rows, columns
and number of nonzero terms: ");
```

```

scanf( "%d%d%d", &num_rows, &num_cols,
        &num_terms );
num_heads =
(num_cols>num_rows)? num_cols : num_rows;
/* set up head node for the list of head
   nodes; upper left corner (左上角)*/
node = new_node();      node->tag = entry;
node->u.entry.row = num_rows;
node->u.entry.col = num_cols;

if (!num_heads) node->right = node;
else { /* initialize the head nodes */
    for (i=0; i<num_heads; i++) {
        temp= new_node();
        hdnode[i] = temp;
        hdnode[i]->tag = head;
        hdnode[i]->right = temp;
        hdnode[i]->u.next = temp;
    }
}

```

$O(\max(n,m))$



```

current_row= 0;    last= hdnode[0];
/*last node in current row*/
for (i=0; i<num_terms; i++)
{
    printf("Enter row, column and value:");
    scanf("%d%d%d", &row, &col, &value);
    if (row>current_row) { /*close current row*/
        last->right= hdnode[current_row];
        current_row= row;  last=hdnode[row];
    }
    MALLOC(temp, sizeof(*temp));
    temp->tag=entry;
    temp->u.entry.row=row;
    temp->u.entry.col = col;
    temp->u.entry.value = value;
    last->right = temp; /*link to row list */
    last= temp;
    /* link to column list */
    hdnode[col]->u.next->down = temp;
    hdnode[col]->u.next = temp;
}

```

利用next field 存放column的last node ←

```

    /*close last row */
    last->right = hdnode[current_row];
    //current_row =row;
    /* close all column lists */
    /*將此列每一個行串列最後一節點指向同一串列的第一個節點*/
    for (i=0; i<num_cols; i++)
        hdnode[i]->u.next->down = hdnode[i];
    /* link all head nodes together */
    /*將所有標頭節點串連起來*/
    for (i=0; i<num_heads-1; i++)
        hdnode[i]->u.next = hdnode[i+1];
    hdnode[num_heads-1]->u.next= node;
    node->right = hdnode[0];
}
return node;
}

```

$O(\max\{\#\_rows, \#\_cols\} + \#\_terms)$

# Write out a Matrix

```
void mwrite(matrix_pointer node)
{ /* print out the matrix in row major form */
  int i;
  matrix_pointer temp, head = node->right;
  printf("\n num_rows = %d, num_cols= %d\n",
        node->u.entry.row,node->u.entry.col);
  printf("The matrix by row, column, and
        value:\n\n");
  for (i=0; i<node->u.entry.row; i++) {
    for (temp=head->right;temp!=head;temp=temp->right)
      printf("%5d%5d%5d\n", temp->u.entry.row,
            temp->u.entry.col, temp->u.entry.value);
    head= head->u.next; /* next row */
  }
}
```

#\_rows

#\_terms

$O(\#\_rows + \#\_terms)$

## Erase a Matrix

```
void merase(matrix_pointer *node)
{
    int i, num_heads;
    matrix_pointer x, y, head = (*node)->right;

    /*free the entry and header nodes by row*/
    for (i=0; i<(*node)->u.entry.row; i++) {
        y=head->right;
        while (y!=head) {
            x = y;  y = y->right;  free(x);
        }
        x= head;  head= head->u.next; /* next row */
        free(x);
    }
    /*free remaining header nodes*/
    y = head;
    while (y!=*node) {
        x = y;  y = y->u.next;  free(x);
    }
    free(*node);  *node = NULL;
}
```

# Doubly Linked List

Move in **forward** and **backward** direction.

Singly linked list (in one direction only)

How to get the preceding node during deletion or insertion?

Using 2 pointers

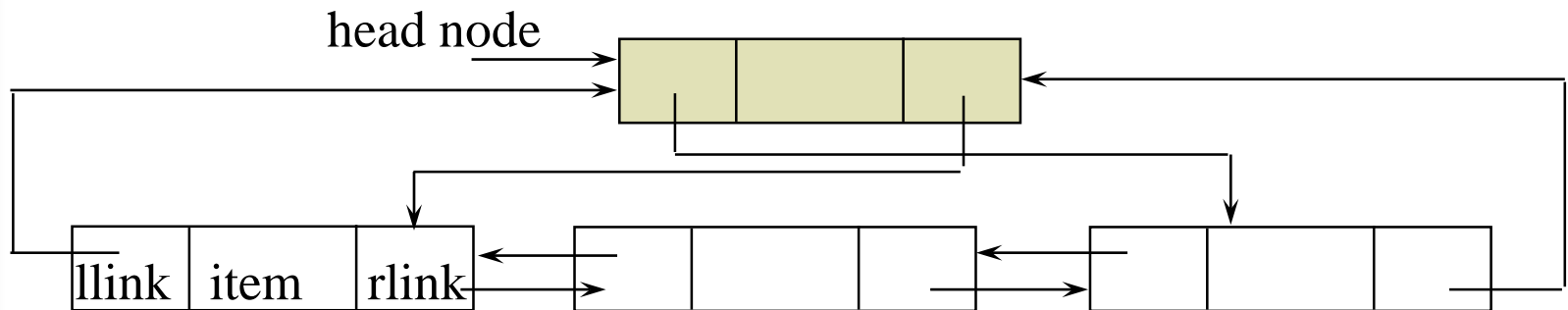
**Node Structure**



# Doubly Linked Lists

```
typedef struct node *node_pointer;  
typedef struct node {  
    node_pointer llink;  
    element item;  
    node_pointer rlink;  
}
```

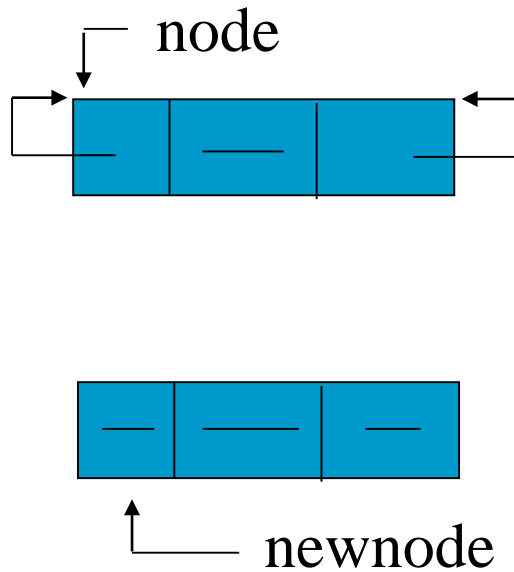
$ptr$   
 $= ptr->rlink->llink$   
 $= ptr->llink->rlink$



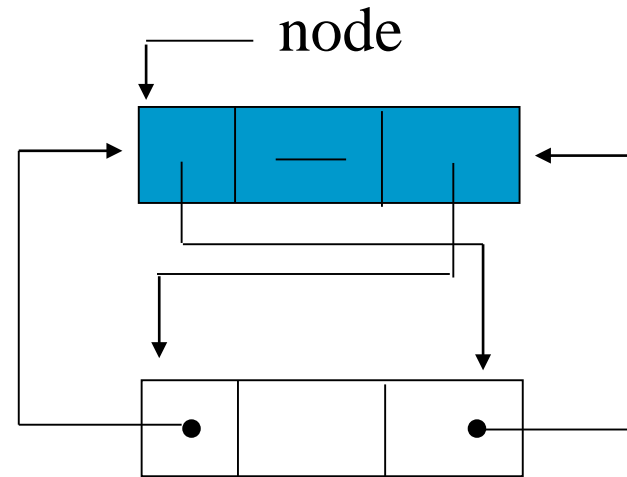


\*Figure 4.22: Empty doubly linked circular list with header node

Before



After

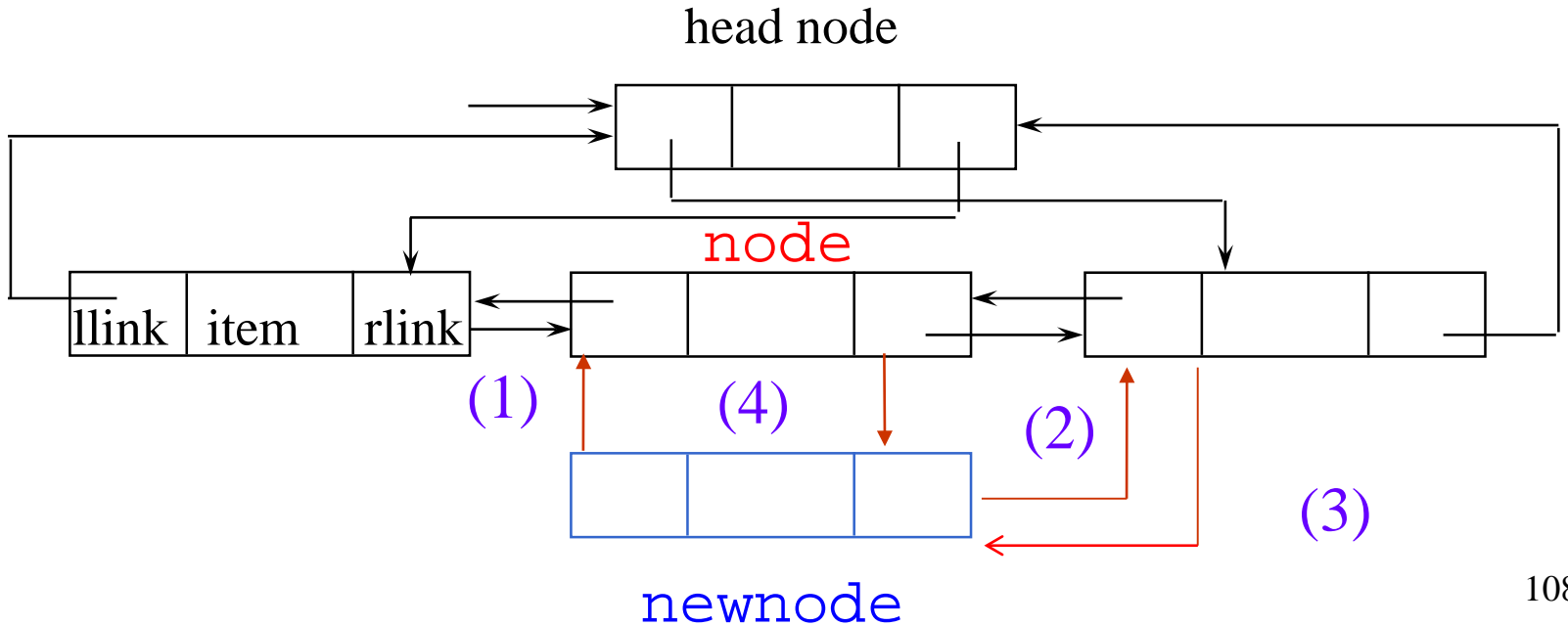


**\*Figure 4.25:** Insertion into an empty doubly linked circular list



# Insert

```
void dinsert(node_pointer node, node_pointer newnode)
{
    (1) newnode->llink = node;
    (2) newnode->rlink = node->rlink;
    (3) node->rlink->llink = newnode;
    (4) node->rlink = newnode;
}
```



# Delete

```
void ddelete(node_pointer node, node_pointer deleted)
{
    if (node==deleted) printf("Deletion of head node
                             not permitted.\n");
    else {
        (1) deleted->llink->rlink= deleted->rlink;
        (2) deleted->rlink->llink= deleted->llink;
        free(deleted);
    }
}
```

